Perspectives of the Muon

Francis Pratt

ISIS Muon Group
Outline of Lecture

• Key properties of the muon
• High energy particle physics perspective
• Radiation physics/chemistry perspective
• Condensed matter physics perspective
• Magnetic resonance perspective
• Chemistry perspective
• Computational science perspective
• μSR experiment perspective
Key Properties of the Muon

Discovered in 1936 by Carl David Anderson and Seth Neddermeyer in California

They used cloud chambers to study cosmic rays

*Carl David Anderson received a Physics Nobel prize the same year for his discovery of the positron using the same technique*

Muon: an unstable charged particle
200 times heavier than the electron

*“Who ever ordered that?”*

*Isidor Isaac Rabi, 1944 Physics Nobel Laureate, commenting after the discovery of the muon*
# Key Properties of the Muon

## Standard Model of Elementary Particles

<table>
<thead>
<tr>
<th>three generations of matter (fermions)</th>
<th>interactions / force carriers (bosons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>III</td>
</tr>
<tr>
<td>u up</td>
<td>gluon</td>
</tr>
<tr>
<td>mass = 2.2 MeV/c² paper</td>
<td>charge = 2/3 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>c charm</td>
<td>top</td>
</tr>
<tr>
<td>mass = 1.26 GeV/c² paper</td>
<td>charge = 2/3 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>t top</td>
<td>gluon</td>
</tr>
<tr>
<td>mass = 173 GeV/c² paper</td>
<td>charge = 2/3 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>d down</td>
<td>photon</td>
</tr>
<tr>
<td>mass = 4.7 MeV/c² paper</td>
<td>charge = -1/2 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>s strange</td>
<td>bottom</td>
</tr>
<tr>
<td>mass = 96 MeV/c² paper</td>
<td>charge = -1/2 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>b bottom</td>
<td>photon</td>
</tr>
<tr>
<td>mass = 4.18 GeV/c² paper</td>
<td>charge = -1/2 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>e electron</td>
<td>Z boson</td>
</tr>
<tr>
<td>mass = 0.511 MeV/c² paper</td>
<td>charge = -1</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>μ muon</td>
<td>W boson</td>
</tr>
<tr>
<td>mass = 105.66 MeV/c² paper</td>
<td>charge = -1</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>τ tau</td>
<td>W boson</td>
</tr>
<tr>
<td>mass = 1.7768 GeV/c² paper</td>
<td>charge = -1</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>νe electron neutrino</td>
<td>Z boson</td>
</tr>
<tr>
<td>mass &lt; 2.2 eV/c² paper</td>
<td>charge = 0</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>νμ muon neutrino</td>
<td>W boson</td>
</tr>
<tr>
<td>mass &lt; 0.17 MeV/c² paper</td>
<td>charge = 0</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>ντ tau neutrino</td>
<td>W boson</td>
</tr>
<tr>
<td>mass &lt; 18.2 MeV/c² paper</td>
<td>charge = 0</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
<tr>
<td>H higgs</td>
<td>H higgs</td>
</tr>
<tr>
<td>mass = 124.87 GeV/c² paper</td>
<td>charge = 2/3 paper</td>
</tr>
<tr>
<td>charge = 1/2</td>
<td>spin = 1/2 paper</td>
</tr>
</tbody>
</table>
### Key Properties of the Muon

Comparison with the more familiar particles: electrons and protons

<table>
<thead>
<tr>
<th></th>
<th>Spin</th>
<th>Mass</th>
<th>Magnetic moment</th>
<th>Charge</th>
<th>Lifetime</th>
<th>Character</th>
<th>Magnetic resonance technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>$\frac{1}{2}$</td>
<td>$m_e$</td>
<td>657 $\mu_p$</td>
<td>-e</td>
<td>$\infty$</td>
<td>electron</td>
<td>ESR</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\frac{1}{2}$</td>
<td>207 $m_e$</td>
<td>3.18 $\mu_p$</td>
<td>+e, -e</td>
<td>2.2 $\mu$s</td>
<td>‘light proton’ ‘heavy electron’</td>
<td>$\mu^+$ SR, $\mu^-$ SR</td>
</tr>
<tr>
<td>p</td>
<td>$\frac{1}{2}$</td>
<td>1836 $m_e$</td>
<td>$\mu_p$</td>
<td>+e</td>
<td>$\infty$</td>
<td>proton</td>
<td>NMR</td>
</tr>
</tbody>
</table>
The standard muon decay process produces a positron or electron and two neutrinos:

\[
\begin{align*}
\mu^+ & \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\
\mu^- & \rightarrow e^- + \bar{\nu}_e + \nu_\mu
\end{align*}
\]

The muon decay is a result of the weak nuclear force

→ Slow decay rate
→ Asymmetric positron/electron emission from parity-symmetry violation

Taking a positive muon

\[ W(\theta) = 1 + a \cos \theta \]

at the maximum positron energy

averaged over all positron energies
Cosmic rays provided the first source of high energy particles

Primary cosmic rays are high energy protons and atomic nuclei mainly from outside the solar system

In the upper atmosphere the primary cosmic rays produce showers of secondary particles including muons, with a large number of the muons reaching the surface of the Earth

Muons played a significant role in the early development of particle physics
Muons provided the first experimental proof of *relativistic time dilation*

Rossi and Hall (1941)

They measured the survival probability of cosmic ray muons between Echo Lake (3240m) and Denver (1616m):

- momentum 440 to 580 MeV/c: 0.70
- momentum > 580 MeV/c: 0.88

Slower decay rate for higher momentum *as predicted by relativity*

Their estimate of the muon lifetime before time dilation effects was 2.4(3) μs

*B. Rossi and D.B. Hall, Phys. Rev. 59, 223 (1941)*
High Energy Particle Physics Perspective

Large Hadron Collider at CERN: the muon tracking detectors Atlas and CMS are used to study the Higgs

Higgs decay events (muon tracks in red)

The LHC-CMS ‘Compact Muon Solenoid’
High precision muon $g - 2$ experiments to test QED and the standard model
Currently 3 -4 $\sigma$ deviation between theory and experiment for the muon gyromagnetic ratio - no new discoveries from LHC are able to explain this difference

New experiments:

Fermilab in the USA (now taking data)

J-PARC in Japan (building up)
High Energy Particle Physics Perspective

Plans being developed for future high intensity muon sources:

1) Muon colliders
   (to reach higher energy concentration than LHC)

2) Neutrino factories
   (to send neutrino beams to neutrino detectors such as Super-Kamiokande in Japan and Gran Sasso in Italy)

Both of these require muon storage rings

Focus on high energy muons
3.9 MeV muon beam stopped and thermalized muons in the sample

3.9 MeV (surface muons)

Muonium (Mu) muon analogue of the H atom

\[ \text{Mu} = \mu^+ + e^- \]

Diamagnetic Final States

- e.g.
- \( \mu^+ (\text{Mu}^+) \)
- Mu\(^-\)

Paramagnetic Final States

- e.g.
- Mu\(^*\)
- Mu-R\(^*\)

- \( kT \)
Atomic ionisation and excitation energy loss to below 35 keV

Radiolytic e\(^-\)

\(\mu^+\) 13.5 eV Mu

\(\mu^+\) e\(^-\) capture

\(\mu^+\) e\(^-\) loss

Thermal Mu

Paramagnetic

Charge exchange cycle

Delayed Mu formation

Chemical reaction

Thermal \(\mu^+\) Diamagnetic

Ionization/reaction

Mu Radical

Paramagnetic

hot (epithermal) reactions are also possible

The thermalisation occurs on a ns time scale and preserves the 100% muon polarisation
Condensed Matter Physics Perspective

Thermalised muons acting as local probes of static and dynamic magnetism

Static magnetism:

1) Muon as a local probe with a limited set of well-defined sites
   
   data analysed in terms of specific sites
   
   complementary site calculations to aid the analysis

2) Muon as a statistical sampling probe
   
   superconducting vortex lattices
   
   magnetic systems with a large cell
   and many stopping sites
   
   systems with a high degree of disorder
   
   quantum delocalised systems
Sensitivity to dynamics:

*Depends on the strength of the magnetic coupling $A$ and the characteristics of the muon source*
μSR dynamics compared with other techniques:

- Remanence
- Ac susceptibility
- NMR
- μSR
- Mössbauer
- Neutrons
Dynamics from ionic motion:

Muon motion as an analogue of hydrogen motion, e.g. in Cu

Motion of other ions can be studied if the muon is relatively immobile, e.g. Li in battery materials

Benefits of μSR versus bulk magnetic studies:

Volume-averaged Probe:
- immune to low concentration impurity phases
- phase segregated samples can easily be identified

Good Sensitivity to antiferromagnetism

Measurements possible in zero applied field
Other Key Features of $\mu$SR

High sensitivity: very small moments can be detected, 0.01 $\mu_B$ or less, e.g. when searching for breaking of time-reversal symmetry in superconductors.

No need to apply an external magnetic field.

No need to align a sample array when making ZF studies.

Excellent compatibility with very low temperatures, since both incoming muon and outgoing positrons easily penetrate the walls and windows of cryostats.
<table>
<thead>
<tr>
<th>Spin</th>
<th>Mass</th>
<th>Magnetic moment</th>
<th>Charge</th>
<th>Lifetime</th>
<th>Character</th>
<th>Magnetic resonance technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>$\frac{1}{2}$</td>
<td>$m_e$</td>
<td>657 $\mu_p$</td>
<td>-e</td>
<td>$\infty$</td>
<td>electron</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\frac{1}{2}$</td>
<td>207 $m_e$</td>
<td>3.18 $\mu_p$</td>
<td>+e</td>
<td>2.2 $\mu$s</td>
<td>‘light proton’</td>
</tr>
<tr>
<td>p</td>
<td>$\frac{1}{2}$</td>
<td>1836 $m_e$</td>
<td>$\mu_p$</td>
<td>+e</td>
<td>$\infty$</td>
<td>proton</td>
</tr>
</tbody>
</table>

$\mu^+$SR as a more sensitive version of $^1$H-NMR
Advantages of μSR over NMR

100% spin polarisation for the muon compared to very weak polarisation for NMR nuclei.

The muon has a larger gyromagnetic ratio than any nucleus.

No need for specific nuclear isotopes, muons can be implanted in anything.

Full field range available for measurements from ZF up to 10 T.

Easy sample set-up and measurement (if you have access to a muon source).

Compatible with low temperature sample environments (e.g. helium dilution refrigerators).

Potential access to a very wide range of correlation times.
Magnetic Resonance Perspective

Disadvantages of $\mu$SR compared to NMR

The muon lifetime limits the potential for spin manipulation by RF methods

The muon stopping sites need to be determined to fully interpret the data

There will be some perturbation of the system under study by the presence of the muon

Larger sample mass is needed
The H atom is the simplest free radical and is highly reactive.

In chemical terms muonium provides a radioactively labelled light isotope of H.

Studying Mu allows the time dependence of its chemical reactions to be studied.
Addition of Mu to a benzene molecule to form a radical:

Distribution of spin density (%)
The isotopic sequence D, H can be extended to D, H, Mu

The low mass of Mu leads to large quantum zero point energy so that Mu probes upper reaches of the interatomic potential energy surface (PES) and can test reaction theory.

Vibrational excitation (e.g. with a laser) takes the muon even higher up the PES ($3 \times$ higher)
Spectroscopic study of muoniated radicals gives information about the molecular spin distribution, molecular environment and molecular dynamics.

The main spectroscopic technique is called Avoided Level Crossing (ALC).

**ALC spectrum for Corannulene**

![Graph showing ALC spectrum for Corannulene with labeled peaks R1, R2, R3, and R4. The graph compares measurements at 40 K and 410 K with corresponding molecular structures for each peak.]

*M. Gaboardi et al, Carbon (2019)*
The implanted muon problem is a variation on the hydrogen interstitial defect problem.

Computational methods such as Density Functional Theory (DFT) can be used to calculate the structure of the interstitial muon defect including the surrounding atomic relaxation.

Extra challenge from the strong quantum character of the muon

Standard computational methods such as DFT give the classical site
Muon Spectrometer

A muon spectrometer has an array of positron detectors placed around the sample to detect the asymmetry

The signal for detector $d$ in a transverse magnetic field takes the form:

$$N^d(t) = N^d_0 \exp(-t/\tau_\mu) \left[ 1 + a_0 P^d_x(t) \right]$$

where $a_0$ is the initial asymmetry and $P^d_x(t)$ is the oscillatory polarisation function

$$P^d_x(t) = \cos(\phi_d + \gamma_\mu B_{TF} t) \ G_x(t)$$

where $\phi_d$ is the detector phase and $G_x(t)$ is the transverse relaxation function
The Muon Instrument

Configuration of detectors and magnetic fields

Note that F and B detectors are defined logically in terms of the initial spin direction.

Instruments often define their detector names in terms of the physical beam geometry.
μSR Experiment Perspective

It is convenient to normalise out the muon decay by working with antiphase detector pairs

\[ A(t) = a_0 P_{FB}(t) = \frac{[N_F(t) - \alpha N_B(t)]}{[N_F(t) + \alpha N_B(t)]} \]

here \( \alpha \) is a calibration factor to correct for any imbalance in the \( F \) and \( B \) detectors

TF Mode

\[ \frac{A(t)}{a_0} = G_x(t) \cos(\gamma_\mu B_T \tau_t) \]

\( G_x(t) \) is the transverse relaxation function

ZF/LF Mode

\[ \frac{A(t)}{a_0} = G_z(t) \]

\( G_z(t) \) is the longitudinal relaxation function

Muon spectroscopy involves measuring \( A(t) \) and analysing it in terms of \( G_x(t) \) and \( G_z(t) \)
• Muons and particle physics: a rich history and continued current relevance

• Muons can easily be implanted into materials to provide a local spin probe

• Extremely wide range of application for muon spectroscopy

• Many advantages of muons over other experimental techniques

• Relatively easy to obtain μSR data on samples of interest, but...

• Data analysis and interpretation often needs a range of knowledge and skills

• Hence this School, whose purpose is to help with this process...
Back to Isidor Rabi for some last words

Did you ask a good question today?

(Isidor Rabi's mother)