Lecture 3
More on magnetism

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Various animals attempting to follow a scaling law.
Some interesting results from magnetism

1) Dynamics in a magnetic metal

2) The spin glass

3) Quantum phase transition
Muon spin relaxation

spin antiparallel to momentum  \( \mu^+ \) decays at rest  \( A(t) \propto \text{Polarization} \)
How (strong collision) dynamics works

**Slow fluctuations**

**Fast fluctuations**

Exponential relaxation
Relaxation from dynamics: longitudinal fields

\[ P_z(t) = \exp(-\lambda t) \]

Polarization \hspace{5cm} Relaxation

\[ \lambda = \frac{\gamma^2}{2} \int_0^{\infty} \int_0^\infty \text{d} \tau \cos \omega_0 \tau \left[ \Phi_{xx}(\tau) + \Phi_{yy}(\tau) \right] \]

\[ \omega_0 = \gamma \mu B_0, \text{ where } B_0 \text{ is the applied longitudinal field} \]

In this sense we use \( B_0 \) to probe dynamics
Dynamics are controlled by a correlation function

\[ \Phi_{ij}(\tau) = 2\gamma_{\mu}^2 \langle B_i(\tau) B_j(0) \rangle \]

\(B_i(t)\) is the fluctuating field at the muon site

The correlation function describes the amplitude and spread of the fluctuating fields
Part I: Magnetism in metals
Observation of the \((T-T_c)/T\) law in MnSi

\[
\lambda \propto \frac{T}{T - T_c}
\]

Electronic behaviour of metals

The distribution of electrons

Excitation spectrum (RPA version)

(a) Excitation spectrum (RPA version)

(b) Electron-hole excitations

(c) Plasmons
Random-phase approximation

\[ p - q \quad k + q \]

Interactions

\[ p \quad q \quad k \]

The bubble approximation

\[ \cdots \]

The random-phase approximation

\[ \frac{1}{1 - \cyclic{\bigoplus}} = (\cyclic{\bigoplus})^{-1} \]

Quantum Field Theory for the Gifted Amateur, OUP (2014)
Random phase approximation

\[ \chi_0(q, \omega) = \sum_p \frac{n_{p+q} - n_p}{E_{p+q} - E_p - \hbar \omega - i\varepsilon} \]

This is a good start, but won’t give you magnetism

The more complicated versions share the same spirit…

Quantum Field Theory for the Gifted Amateur, OUP (2014)
How we do better

1) Use a Hubbard model

\[ \hat{H} = \hat{T} + U \sum_j \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} \]

2) Include a self-consistent renormalization by coupling modes together

The self-consistent result:

\[ \chi(q, \omega) = \frac{\chi(q) \Gamma(q)}{\Gamma(q) - i\omega} \]

(This resembles a harmonic oscillator susceptibility)
Correlation/response

- Correlation function

\[ G_{ij}(x, t, x', t') = \langle M_i(x, t) M_j(x', t') \rangle \]

- Response function

\[ \delta M_i(x, t) = \int dt' d^3x' \chi(x, t, x', t') \delta H_i(x', t') \]

We often deal with the Fourier transforms of these quantities.
How we do better

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The muon response

Zero-field response

\[ \lambda = g \sum_i \int_0^\infty dt \langle B^i(0)B^i(t) \rangle \]

Relate this to the spins

\[ \lambda = g \sum_q |D(q)|^2 \int_0^\infty dt \langle S(-q,0)S(q,t) \rangle \]
The muon response

Zero-field response

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Relate this to the spins

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Fluctuation dissipation theorem

\[ \tilde{G}(q, \omega = 0) = 2k_B T \lim_{\omega \to 0} \frac{\chi''(q, \omega)}{\omega} \]

The result

\[ \lambda = 2gk_B T \lim_{\omega \to 0} \sum_q |D(q)|^2 \frac{\chi''(q, 0)}{\omega} \]

We therefore have a relaxation in terms of a susceptibility
Self consistent renormalization and muons

\[ \chi(q, \omega) = \frac{\chi(q) \Gamma(q)}{\Gamma(q) - i\omega} \]

Muon response

\[ \lambda = 2g k_B T \lim_{\omega \to 0} \sum_q |D(q)|^2 \frac{\chi''(q, 0)}{\omega} \]

Assuming a simple coupling

\[ \lambda \propto g k_B T \sum_q \frac{\chi(q)}{\Gamma(q)} \]
Four assumptions \((T > T_c)\)

Linewidth for a ferromagnet

\[
\Gamma(q) = \frac{\gamma q}{\chi(q)}
\]

Orstein-Zernike susceptibility

\[
\chi(q) = \frac{\chi_0}{1 + q^2 / \kappa^2}
\]

Curie-Weiss susceptibility

\[
\chi_0 = \frac{C}{(T - T_c)}
\]

Inverse coherence length

\[
\frac{1}{\xi} = \kappa = \kappa_0 \left( \frac{|T - T_c|}{T_c} \right)^{\frac{1}{2}}
\]
Stages of the computation

\[ \lambda \propto g k_B T \int \frac{dq \, q \chi_0^2}{(1 + q^2/\kappa^2)^2} \]

Integrating up to a maximum \( q_{\text{max}} \)

\[ \lambda \propto g k_B T \frac{\chi_0^2 q_{\text{max}}^2}{1 + q_{\text{max}}^2/\kappa^2} \]
The result

$$\lambda \propto g k_B T \chi^2_0 \kappa^2$$

$$\propto T \left( \frac{1}{T - T_c} \right)^2 \left( \frac{|T - T_c|}{T_c} \right)$$

$$\propto \frac{T}{T - T_c}$$

Conclusion: we can make links to sophisticated theoretical treatments using the tools from dynamics
Observation of the \((T-T_c)/T\) law in MnSi

Magnetism in spin glasses

Examples: CuMn and AuFe at 1% conc.
The many faces of magnetism
Magnetism in spin glasses

Relaxation follows a root exponential form

\[ P(t) = \exp\left[-(\lambda t)^{1/2}\right] \]

Magnetism in spin glasses

Model of a spin glass

Magnetic ions dissolved in a metallic host

This state has interesting dynamics we would like to capture
An exact result: the field distribution is Lorentzian

\[ p^L(B_i) = \frac{\gamma \mu}{\pi} \left( \frac{a}{a^2 + \gamma^2 \mu B_i^2} \right) \]

\[ a \propto \text{number of magnetic spins} \]

Average over this and we obtain the Lorentzian Kubo-Toyabe function

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} (1 - at)e^{-at} \]

Dynamicizing this leads to unobserved behaviour
Muons in a dilute environment

The range of fields accessible changes between different muon sites
Muons in a dilute environment

The range of fields accessible changes between different muon sites

Assume each range has a Gaussian distribution

\[ p^G(B_i, \Delta_j) = \left( \gamma_\mu^2 / 2\pi\Delta_j^2 \right)^{1/2} e^{-\gamma_\mu^2 B_i^2 / 2\Delta_j^2} \]

Each site relaxes according to a KT

\[ P^G_z(t, \Delta_j) = \frac{1}{3} + \frac{2}{3}(1 - \Delta_j^2 t^2)e^{-\frac{\Delta_j^2 t^2}{2}} \]
Typical spectra for polycrystalline samples

Case I:
static order

Case II:
static disorder

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma B \mu t) \]

\[ P_z(t) = \frac{1}{3} + \frac{2}{3} \left( 1 - \Delta^2 t^2 \right) \exp\left( -\frac{\Delta^2 t^2}{2} \right) \]
Muons in a dilute environment

The range of fields accessible changes between different muon sites

Assume each range has a Gaussian distribution

\[ p^G(B_i, \Delta_j) = \left( \frac{\gamma_\mu^2}{2\pi \Delta_j^2} \right)^{\frac{1}{2}} e^{-\gamma_\mu^2 B_i^2 / 2\Delta_j^2} \]

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\[ P^G_z(t, \Delta_j) = \frac{1}{3} + \frac{2}{3} \left( 1 - \Delta_j^2 t^2 \right) e^{-\frac{\Delta_j^2 t^2}{2}} \]
Using this model, we can recreate the Lorentzian distribution if we pick a distribution of Gaussians appropriately.

\[ p^L(B_i) = \int_0^\infty \mathrm{d}\Delta_j \, p^G(B_i, \Delta_j) \rho(\Delta_j). \]
Using this model, we can recreate the Lorentzian distribution if we pick a distribution of Gaussians appropriately

\[ p^L(B_i) = \int_0^\infty d\Delta_j \rho^G(B_i, \Delta_j) \rho(\Delta_j). \]

If we pick

\[ \rho(\Delta_j) = \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \left( \frac{\Delta_j}{\Delta_j^2} \right) e^{-\frac{a^2}{2\Delta_j^2}} \]

then we correctly find

\[ p^L(B_i) = \frac{\gamma_\mu}{\pi} \left( \frac{a}{a^2 + \gamma_\mu^2 B_i^2} \right) \]
The dynamicization can now be carried out, using the prescription

\[
P^L_z(t, a, \nu) = \int_0^\infty P^G_z(t, \Delta_j, \nu) \rho(\Delta_j) d\Delta_j
\]

Dynamic Gauss Distribution

Recall that in the fast-fluctuation limit we have the Gaussian result

\[
P^G_z(t, \Delta_j, \nu) = e^{-\frac{2\Delta_j^2}{\nu} t}
\]

Plugging in, we have,

\[
P^L_z(t, a, \nu) = e^{-\left(\frac{4a^2 t}{\nu}\right)^{\frac{1}{2}}}
\]

This is stretched-exponential behaviour
How (strong collision) dynamics works

Slow fluctuations

Exponential relaxation

Fast fluctuations
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Predictions of the model

\[ P_z^{L}(t, a, \nu) = \int_{0}^{\infty} P_z^{G}(t, \Delta_j, \nu) \rho(\Delta_j) d\Delta_j \]

Limiting case

\[ P_z^{L}(t, a, \nu) = e^{-\left(\frac{4a^2}{\nu}t\right)^{\frac{1}{2}}} \]
Magnetism in spin glasses

Part III: introduction to quantum magnetism
Systems with energy gaps
Spin Peierls: another fate for 1D spin systems

Uniform spin system

Dimerized spin system below $T_{sp}$
Dimers have an $S=0$ ground state (no magnetization) and a gap to the first excited magnetic state.
Spin Peierls: another fate for 1D spin systems

MEM(TCNQ)$_2$

S.J. Blundell et al., JPCM 9 L119 (1997)
Quantum magnetism and dimers

$[\text{Cu(gly)(pyz)}](\text{ClO}_4)$

[Cu(gly)(pyz)](ClO$_4$)

Bleaney-Bowers Susceptibility: $J=7.5$ K
[Cu(gly)(pyz)][ClO_4]  

Bleaney-Bowers Susceptibility: \( J = 7.5 \) K

No order in ZF down to 30 mK
Isolated dimers
Weakly coupled dimers

In an idealized case we expect a quantum phase transition to $XY$ magnetic order

Weakly interacting dimers

In an idealized case we expect a quantum phase transition to $XY$ magnetic order
Weakly coupled dimers

In an idealized case we expect a quantum phase transition to $XY$ magnetic order.
[Cu(gly)(pyz)](ClO₄)

Bleaney-Bowers
Susceptibility: $J=7.5$ K

Two set of transitions in applied field
[Cu(gly)(pyz)](ClO$_4$)$_2$

Consistent with a FM Coupled dimers

Suggests $J = 7.3$ K and $J' = 3.3$ K

How do we understand the occurrence of magnetic order?
- Phase transitions
[Cu(gly)(pyz)](ClO$_4$)$_2$

Consistent with a FM Coupled dimers

Suggests $J = 7.3$ K and $J' = 3.3$ K

Conclusions

Magnetic metals are complicated, but can be treated at various levels of approximation.

The muon relaxation close to the transition can be accounted for using Moriya’s self-consistent theory.

Spin glass relaxation requires an average over a distribution of fluctuating Gaussian distributions: it gives a stretched exponential.

Muons can be used for many more systems.