Muons in superconductors

• Lesson I – the land we are exploring
  – Introduction: superconductivity, a story of three length-scales
  – London equations and the penetration depth
  – Ginzburg Landau equations and the coherence length
• Lesson II – the workhorse of $\mu$SR
  – The Abrikosov flux lattice
  – Muon determination of the penetration depth
  – Conventional and unconventional superconductivity: a glance
  – BCS: the gap and its temperature dependence
• Lesson III – material science
  – Clean vs. dirty superconductors
  – A phase diagram for superconducting materials
  – Towards atomic scale coherence: nanoscopic coexistence
  – Triplet superconductivity, topological superconductivity (?)
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  – Clean vs. dirty superconductors, extreme type II
  – A phase diagram for superconducting materials
  – Towards atomic scale coherence: nanoscopic coexistence
  – Triplet superconductivity, topological superconductivity (?)
Introduction: Superconductivity

1911 Heike Kamerlingh Onnes

19/08/2019
Advanced School on Muon Spectroscopy

DEPARTMENT OF MATHEMATICAL, PHYSICAL & COMPUTER SCIENCES, PARMA
Elemental superconductors

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Lanthanide Series
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- 60 Nd
- 61 Pm
- 62 Sm
- 63 Eu
- 64 Gd
- 65 Tb
- 66 Dy
- 67 Ho
- 68 Er
- 69 Tm
- 70 Yb
- 71 Lu

Actinide Series
- 90 Th
- 91 Pa
- 92 U
- 93 Np
- 94 Pu
- 95 Am
- 96 Cm
- 97 Bk
- 98 Cf
- 99 Es
- 100 Fm
- 101 Md
- 102 No
- 103 Lr
Elemental superconductors

Different sources: different shades of optimism

http://www.superconductors.org>Type1.htm
Why superconductors?

10 T conventional solenoid:
5 000 A in 1600 turns
5 MW homes in Abingdon

CERN
LHC
1232 main dipole
392 quadrupole
6000 corrector magnets

30 l liquid He/month
Why superconductors?

ITER tokamak

MRI
Why superconductors?

Quantum computation: transmons

\[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \]
Why superconductors?

A rare example of macroscopic quantum coherent state (with superfluids) $|\psi\rangle$

Also metals are an example of macroscopic quantum coherent state. $|\mathbf{k}\rangle$

However

\[ \rho = \frac{m}{ne^2\tau} \]
\[ \tau \approx 10^{-15} \text{ s} \]

\[ \rho = 0 \]
\[ \tau^* \approx 10^{17} \text{ s} \]

*decays in $10^{10}$ years (not the same as Drude $\tau$!)}
Why superconductors
Persistent currents – 1
Perfect diamagnet

Shielding in three steps: 1 → 2 → 3

1 - No field
\[ \Phi_B, \Gamma = 0 \]
\[ T > T_c \]

2 - Zero Field cooling
\[ \Phi_B, \Gamma = 0 \]
\[ T < T_c \]

3 - Turn field on
\[ I_{\text{ext}} \]
\[ \Phi_B, \Gamma = 0 \]
\[ T < T_c \]

screening eddy-currents to keep

This happen also in a superconductor
\[ \rho = 0 \]
Persistent currents – 2
Perfect diamagnet

Establishing a persistent currents in three steps: 1 → 2 → 3

1 – Turn field on above $T_c$

$\Phi_{B,\Gamma} \neq 0$

$T > T_c$

2 – Field cooling

$\Phi_{B,\Gamma} \neq 0$

$T < T_c$

This does not happen in a superconductor

3 – Turn field off

$\frac{d\Phi_B}{dt} = 0$

$T < T_c$

$\Phi_{B,\Gamma} \neq 0$

In a perfect conductor $\rho = 0$
Meissner-Ochsenfeld effect: $1 \rightarrow 2$

1 - Set field above $T_c$
\[
\Phi_{B,\Gamma} \neq 0
\]

2 - Field cooling
\[
\Phi_{B,\Gamma} = 0
\]

The flux is expelled, so the real rule is

This would \textbf{not} happen to a perfect conductor

Summary
A superconductor in an external field $\mathbf{B}$, both F cooling and ZF cooling, expels the flux $\Phi_{B,\Gamma}$.
Field Cooling vs Zero Field Cooling

Negative M/H for ZFC is also the response of a perfect conductor

Negative M/H in FC is the signature of superconductivity

Extrinsic difference due to flux pinning

Zhao et al. PNAS 116 12156
Type I and Type II superconductors

Critical field: superconductivity disappears for $H > H_c$

Type I:

$\chi = \left. \frac{dM}{dH} \right|_{H=0} = -1$

Type II:

$|\chi| = \left. \frac{dM}{dH} \right|_{H=0} \ll 1$

Jing Guo et al. PNAS 114, 13144

$(\text{TaNb})_{0.67} (\text{HfZrTi})_{0.33}$
Type I and Type II

What inhomogeneity for $H > H_{c1}$?

(super)current vortices encircling quantized magnetic flux $\Phi_0 = \frac{h}{2e}$

Vortices in YBCO imaged by scanning SQUID microscopy
Three length-scales

• London penetration depth
  – $\lambda$ controls the magnetic field penetration

• Coherence length
  – $\xi$ controls the quantum coherence of the ground state

• Mean free path
  – $\ell$ controls scattering
London equation

Sketch of deep argument on electron wavefunction:

- incoherent in normal Drude metal: \( \mathbf{J} = n q \mathbf{v} \)

No power supply

\[
\langle p \rangle = 0 \quad \rightarrow \quad m \langle v \rangle = 0 \quad \rightarrow \quad \langle J \rangle = 0
\]

- quantum coherent in superconductors

Superconducting state: \( \langle p \rangle = 0 \) even after switching fields on.

Minimal substitution: \( m \mathbf{v} = \mathbf{p} - e \mathbf{A} \)

\[
\langle v \rangle = -\frac{e}{m} \mathbf{A} \quad \rightarrow \quad \mathbf{J}_s = -\frac{n e^2}{m} \mathbf{A}
\]

London equation

Fritz London, 1900-1956
London penetration depth

\[ J_s = -\frac{ne^2}{m} A \]

London equation

Substituting in Ampère law one obtains

\[ \nabla^2 B = \frac{\mu_0 ne^2}{m} B \]

For London

- electron mass
- electron density
- electron charge

\[ \lambda = \left( \frac{m}{\mu_0 ne^2} \right)^{\frac{1}{2}} \]

London penetration depth

After Cooper pairs

Also

\[ \nabla^2 A = \frac{\mu_0 ne^2}{m} A \]
London penetration depth derivation

\[ J_s = -\frac{ne^2}{m} A \]

London equation

take the curl of the stationary Ampère law \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)

Vector identity
\[
\nabla \times \nabla \times \mathbf{B} = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \left(-\frac{ne^2}{m} \nabla \times A\right)
\]

By Gauss law \( \nabla \cdot \mathbf{B} = 0 \)

\[
\lambda = \left(\frac{m}{\mu_0 ne^2}\right)^{\frac{1}{2}}
\]

\[
\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}
\]

Magnetic field (London approximation)
What does $\lambda$ imply?

Ampere law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

Guess the solution

$$\mathbf{B} = B_0 e^{-y/\lambda} \hat{z}$$

Right!

$$\nabla \times \mathbf{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & B_0 e^{-y/\lambda} \end{vmatrix} = -\frac{B_0}{\lambda} e^{-y/\lambda} \hat{x}$$

Thin sample

$$\mathbf{B}(x, y, z) = \mu_0 k_0 \frac{\cosh(y/\lambda)}{\cosh(d/\lambda)} \hat{z}$$
Exercise: do it properly

Check that

\[
B = \mu_0 k_0 \hat{\hat{z}} \begin{cases} 
1 & y < 0 \\
\frac{e^{-y/\lambda}}{\lambda} & y > 0
\end{cases}
\]

and

\[
A = -\mu_0 k_0 \hat{\hat{x}} \begin{cases} 
y & y < 0 \\
\lambda e^{-y/\lambda} & y > 0
\end{cases}
\]

are solutions of

\[
\nabla^2 B = \frac{1}{\lambda^2} B
\]

and

\[
\nabla^2 A = \frac{1}{\lambda^2} A
\]

with

\[
J_s = -\frac{ne^2}{m} A
\]
Anisotropic metals

\[ \lambda^2 = \frac{m}{\mu_0 ne^2} \rightarrow \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_c \end{bmatrix} \frac{1}{\mu_0 ne^2} \]

\[ \lambda_a^2 = \frac{m_a}{\mu_0 ne^2} \]

\[ \lambda_b^2 = \frac{m_b}{\mu_0 ne^2} \]
LEM experiment

Kiefl et al. Phys. Rev. B. 81 180502

![Graph showing the relationship between energy (keV) and average local field (mT)]

- The graph illustrates the average local field (mT) as a function of energy (keV).
- It demonstrates two distinct orientations: $\vec{j} \parallel a$-axis and $\vec{j} \parallel b$-axis.
- The data points are plotted against energy, with the applied field marked.
- The graph features two lines, one for each orientation, showing the mean depth (nm) on the y-axis and energy (keV) on the x-axis.
**Landau model**

The order parameter is a complex function $\psi$ and the free energy density is

$$f_s(H) = f_n(H) + a(T - T_c)|\psi|^2 + \frac{b}{2} |\psi|^4 + \frac{\mu_0}{2} HM$$

For $a, b > 0$ (only below $T_c$ and below $B_c$)

Condensation energy $\equiv$ maximum energy that supercurrents can expell, corresponds to a tiny free energy density

$$f_n(H_c) - f_s(H_c) = \frac{\mu_0}{2} H_c^2 = \frac{1}{2\mu_0} B_c^2$$

Compare

$$v_{cell} \frac{B_c^2}{2\mu_0} \approx 1 \mu eV$$

$$\epsilon_F \approx 1 eV$$
Ginzburg-Landau coherence length

In zero $B$ field, linearised ($b = 0$)

$$f_s(0) = f_n(0) + a(T - T_c)|\psi|^2 + \frac{\hbar^2}{2m} |\nabla \psi|^2$$

Ginzburg-Landau free energy density

The ratio of the order parameter to the gradient term is a square lengthscale

$$\left[ \frac{|\psi|^2}{|\nabla \psi|^2} \right] = \frac{\hbar^2}{2ma|T - T_c|} = \xi^2$$

energy loss

$\propto B_c^2 \xi$

condensation energy

energy gain

$\propto B_c^2 \lambda$

(shielding)
Type-I vs Type-II again

It is convenient to have normal-superconductor interfaces. The energy gain is

\[ \propto B_c^2 \lambda \]  

(shielding)

Homogeneous superconductor interfaces cost energy

\[ \kappa = \frac{\lambda}{\xi} \]

\[ \kappa < \frac{1}{\sqrt{2}} \]

Type I:

\[ H_c \]

Type II:

\[ H_{c1} < H < H_c \]

Meissner

inhomogeneous

energy gain

\[ \propto B_c^2 \xi \]

energy loss

\[ \propto B_c^2 \xi \]

condensation energy
GL equations

Minimizing the free energy with respect to $\nabla \psi$

$$f = a(T - T_c)|\psi|^2 + \frac{1}{2m^*} \left| \frac{\hbar}{i} \nabla \psi \right|^2$$

GL (linearised) equation

$$-\frac{\hbar^2}{2m^*} \psi = a(T_c - T)\psi$$ : like a Schrödinger equation

In a magnetic field

Minimizing the free energy with respect to $\nabla \psi$, $A$

$$f = a(T - T_c)|\psi|^2 + \frac{1}{2m^*} \left| \left( \frac{\hbar}{i} \nabla + 2eA \right) \psi \right|^2 + \frac{1}{2\mu_0} |\nabla \times A|^2$$

Two GL (linearised) equation

$$\left[ -\frac{\hbar^2}{2m^*} \left( \nabla + \frac{e^*}{\hbar} A \right)^2 + a(T - T_c) \right] \psi = 0$$

$$J_s = -\frac{ne^2}{m} A$$

$$J = -\frac{e^* \hbar}{i2m^*} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{(e^*)^2}{m^*} |\psi|^2 A$$

cfr. London
Single London vortex

It is convenient to have normal-superconductor interfaces when

\[ \kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} \]

May be convenient to have field defects

\[ \nabla^2 B - \frac{1}{\lambda^2} B = \Phi_0 \delta(r) \]

Field must be quantized!

1 fluxon \[ \int_A B \cdot da = \Phi_0 = \frac{\hbar}{2e} = 2.0678 \cdot 10^{-15} \text{ Wb} \]
Next lecture: type II

what does an implanted muon detect?

From a single vortex to a flux lattice to the normal state