Muons in superconductors

- **Lesson I – the land we are exploring**
  - Introduction: superconductivity, a story of three length-scales
  - London equations and the penetration depth
  - Ginzburg Landau equations and the coherence length

- **Lesson II – the workhorse of μSR**
  - The Abrikosov flux lattice
  - Muon determination of the penetration depth
  - Conventional and unconventional superconductivity: a glance
  - BCS: the gap and its temperature dependence

- **Lesson III – material science**
  - Clean vs. dirty superconductors, extreme type II
  - A phase diagram for superconducting materials
  - Towards atomic scale coherence: nanoscopic coexistence
  - Triplet superconductivity, topological superconductivity (??)
Type II superconductors

$B_{c2} = \frac{\Phi_0}{2\pi\xi^2}$

Area = condensation energy density

Vortex (flux line) lattice

found as an ingegnous solution of linearised GL equations

Alexei A. Abrikosov
Nobel Prize 2003
The flux line lattice

Most often triangular (or square) free energy minimum

\[ a_\Delta = b_\Delta = \left( \frac{4}{3} \right)^{\frac{1}{4}} \left( \frac{\Phi_0}{B} \right)^{\frac{1}{2}} \]

incommensurate to crystal lattice (much larger)

Field does not vanish due to overlap

Muons sample uniformly the field distribution \( p(B) \)

TF-\( \mu \)SR asymmetry:

\[ A(t) = A_0 \int p(B) \cos(\gamma_\mu Bt + \phi) dB \]

order parameter vanishes at the core centre
Measure TF asymmetry $A(t)$

$$B = B\hat{z}$$

$S_\mu$

muon beam

ISIS/J-PARC

$$B = B\hat{z}$$

PSI/TRIUMF

away from sample surface

$B(\mathbf{r}) = B(\mathbf{r})\hat{z}$

$$A(t) = A_0 \int p(B) \cos(\gamma_\mu Bt + \phi) dB$$

How do you get $p(B)$ from the asymmetry?

Fourier transform

$$p(B) = p\left(\frac{-\omega}{\gamma_\mu}\right) = \frac{1}{A_0} \int A(t) \cos(\omega t + \phi) dt$$

phase tuning!
Field distribution of a vortex lattice

London model

\[ \nabla^2 B - \frac{1}{\lambda^2} B = \Phi_0 \hat{z} \sum_\mathbf{R} \delta(\mathbf{R}) \]

\[ \mathbf{R} = n \mathbf{a} + m \mathbf{b} \]

\[ \langle B \rangle = \frac{\Phi_0}{|\mathbf{a} \times \mathbf{b}|} \]

Solution by (Fast) Fourier Transform

\[ \mathbf{Q} = n \mathbf{a}^* + m \mathbf{b}^* \]

\[ |\mathbf{a}^*| = \frac{2\pi}{|\mathbf{a}|} \]

\[ B(\mathbf{r}) = \langle B \rangle \sum_\mathbf{Q} b(\mathbf{Q}) e^{i\mathbf{Q} \cdot \mathbf{r}} \]

\[ b(\mathbf{Q}) = \frac{1}{1 + \lambda^2 Q^2} \]
The field distribution by $\mu$SR

\[ B(r) = \langle B \rangle \sum_Q b(Q) e^{iQ \cdot r} \]

\[ b(Q) = \frac{1}{1 + \lambda^2 Q^2} \]

\[ p(B) = \int dS_r \left| \frac{dB}{dx} \right|^{-1} \]

\[ B_q = B(Q) \]
\[ Br = \text{fft}(B_q) \]
\[ pB = \text{histogram}(Br) \]
Textbook case: Nb single crystal

\[
\int p(B) e^{(B'-B)^2/2\sigma^2} \, dB
\]

Gaussian convolution

By eye: very correlated fits!
No chance of fitting \( \lambda, \xi, \sigma, B \) from single \( p(B) \)

Solution: global fits

muons in sample holder

Herlach et al. Hyperfine Interactions 63, 41
Another textbook case: Vanadium

M. Laulajainen  PHYSICAL REVIEW B 74, 054511 (2006)
Details – II : cut-off functions

A cut-off function represents a finite $\xi$
(several approximations quoted in the literature)

Actually London model

$$\nabla^2 B - \frac{1}{\lambda^2} B = \Phi_0 \hat{\sum} \delta(R)$$
approximates GL equations.

Better GL approximation for extreme type II
e.g. Yaouanc et al. Phys. Rev. B 55, 11107

$$B(r) = \langle B \rangle \sum_Q b(Q)e^{iQ \cdot r}$$

$$b(Q) = \frac{e^{-\xi^2 Q^2/2}}{1 + \lambda^2 Q^2}$$

Better approximation for $b(Q)$

$$b(Q) = \frac{uK_1(u)(1-x^4)}{\lambda^2 Q^2}$$

Modified Bessel function

$$K_1$$

$$x = \frac{B}{B_c^2}$$

$$u = \xi Q \left( \sqrt{2} - \frac{0.75}{\kappa} \right) \sqrt{1 + x^4} \sqrt{1 - 2x(1 - x)^2}$$
Details – I : extreme type II

For many high $T_c$ superconductors \( \kappa = \frac{\lambda}{\xi} \geq 10 \)

\[
B(r) = \langle B \rangle \sum_{Q} b(Q)e^{iQ \cdot r} \\
b(Q) = \frac{1}{1 + \lambda^2 Q^2}
\]

with \( x = \frac{B}{B_{c2}} \) we get \( \lambda^2 Q_0^2 \approx 5.4 x \kappa^2 \gg 1 \)

\[
b(Q) \propto \frac{1}{\lambda^2}
\]

In this case TF-μSR provides a simple direct measurement of $\lambda$

\[
\left( \langle B^2 \rangle - \langle B \rangle^2 \right) = \langle B \rangle^2 \sum_{Q \neq 0} b^2(Q) \propto \frac{1}{\lambda^4}
\]

the standard deviation of the μSR lineshape is proportional to $\lambda^{-2}$

\[
\sigma[\mu s^{-1}] = \frac{7.904 \cdot 10^4}{\lambda^2 [\text{nm}]}
\]
Summary

\[ \int p(B) e^{(B' - B)^2 / 2\sigma^2} dB \]

\[ B(r) = \langle B \rangle \sum_Q \frac{C(\xi Q)}{1 + \lambda^2 Q^2} e^{iQ \cdot r} \]

Sometimes it is easier to determine

\[ \langle \Delta B^2 \rangle = \langle B \rangle \sum_{Q \neq 0} b^2(Q) \propto \frac{1}{\lambda^4} \]

\[ \sigma = \gamma\mu \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2} \]

parameters

\( \sigma \) inhomogeneity (disorder)

\( \xi \) physically interesting

\( \lambda \)

\( C \) cut-off function

from a multi-Gaussian fit
Caution: nuclear dipoles

Subtract nuclear width in quadrature!

\[ \sigma_{FL} = \gamma_\mu \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2} \]

\[ \sigma^2 = \sigma_{FL}^2 + \sigma_n^2 \]

FeSe\textsubscript{0.85} Khasanov Phys Rev B 78 220510(R)
Angle dependence in YBa$_2$Cu$_3$O$_{6.9}$

\[ \lambda^2 = \frac{m}{\mu_0 ne^2} \rightarrow \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_c \end{bmatrix} \frac{1}{\mu_0 ne^2} \]

for uniaxial systems

\[ \sigma_{FL}(\theta) = \sigma_{FL}(0) \left[ \cos^2 \theta + \left( \frac{m_a}{m_c} \right) \sin^2 \theta \right]^{1/2} \]

Forgan Hyperfine Interactions 63 41
Cubitt Physica C 213 126

Thiemann Phys Rev B 39 11406
Anisotropic polycrystal

Anisotropy dictated by

\[
[\lambda^2] = \frac{1}{\mu_0 ne^2} \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_c \end{bmatrix}
\]

\[\Gamma = \sqrt{\frac{m_a}{m_c}}\]

Polycrystal average

\[\kappa > 10 \quad \text{(London limit)}\]

\[\sigma[\mu S^{-1}] = \frac{7.904 \cdot 10^4}{\lambda_{\text{eff}}^2 [\text{nm}]}\]

\[\lambda_a = \frac{\lambda_{\text{eff}}}{\sqrt{F_2}} \rightarrow 1.228 \lambda_{\text{eff}}\]

Barford Physica C 156 515
Cuprates: a flash picture

\[ \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \]

\[ T_N \]

optimal doping

underdoping

overdoping

\[ T_g \]

\[ T_c \]

[\text{Sr}] = hole density

Figure Barišić PNAS 110 12235

\[ \text{La}^{3+}_{2-x}\text{Sr}^{2+}_x \text{CuO}_4 \]

\[ x\text{h}^{1+} \]
Indirect fit of coherence length

\[ \sigma = \gamma \mu \sqrt{\langle \Delta B^2 \rangle} = \gamma \mu \langle B \rangle \left( \sum_{Q \neq 0} \frac{e^{-\xi^2 Q^2}}{1 + Q^2 \lambda^2 / (1 - \frac{B}{B_{c2}})} \right) \]

\[ Q(B) = \left( \frac{2\pi}{\alpha} \right)^2 \frac{B}{\Phi_0} \]

From field dependence

\[ B_{c2} \]

Hillier Applied magnetic Resonance 13 95
Temperature dependence of $\sigma$

Two fluid model, for $t = \frac{T}{T_c}$

$$\rho = \frac{t^4}{\rho_n} + 1 - t^4$$

$$\frac{1}{\lambda^2} = \frac{\mu_0 e^2 n_s}{m}$$

$$\frac{\sigma(T)}{\sigma(0)} = \rho_s(T) = \frac{\lambda^2(0)}{\lambda^2(T)}$$

normalised supercarrier density

$$\rho_s(t) = 1 - t^4$$

Caution
- polycrystals
- missing low temperature

Pümpin Phys Rev B 42 8019
A glimpse on BCS

Two main ingredients for BCS

- Attractive $V$ between electrons, mediated by a boson

\[
\chi_{e-ph}(\omega) = \frac{g_{e-ph}^2}{\omega^2 - \omega_D^2}
\]

Weak coupling, adiabatic* limit, a hierarchy of energies

\[\hbar \omega \approx k_B T \ll \hbar \omega_D \ll \epsilon_F\]

- Presence of a Fermi sea

\[
V = (1 + \chi) \frac{e^2}{r} \approx \chi \frac{e^2}{r} < 0
\]

* Migdal theorem
At $T = 0$  $\Delta(0) = 1.764 \, k_B T_c$ and the temperature dependence is

$$\Delta(T) = \Delta(0) \tanh \left( \frac{\pi T_c}{\Delta(0)} \sqrt{\frac{1 - t}{t}} \right)$$

new definition of coherence length  $\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)}$
**ρ(t) beyond two fluid model**

$\rho(t) = \frac{n_s(T)}{n_s(0)} = 1 - \frac{1}{2t} \int_0^\infty \frac{dx}{\cosh^2 \left( \frac{\sqrt{x^2 + \delta^2(t)}}{2t} \right)}$

BCS supercarrier density

e.g. LaO$_{0.5}$F$_{0.5}$BiS$_2$

![Graph showing n_s(T)/n_s(0) vs T[K]](image)

Lamura Phys Rev B 88, 180509(R)
Beyond phonon coupling

BCS weak coupling assumes \( \Delta_k = \Delta \) constant in \( k \) space

Other couplings may have different symmetries

\[
\Delta(\hat{k}) = \Delta(0) g(\hat{k})
\]

<table>
<thead>
<tr>
<th>symmetry</th>
<th>( \Delta(0) )</th>
<th>( g(\hat{k}) )</th>
</tr>
</thead>
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<td>( s )</td>
<td>( 1.763 , k_B T_c )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>( 2.14 , k_B T_c )</td>
<td>( \cos 2\phi )</td>
</tr>
</tbody>
</table>

\[
\Delta(\phi) = 0 \quad \phi = \frac{\pi}{4}(2n + 1)
\]

mediated by spin fluctuations in cuprates

lines of nodes
BCS gap: different coupling

A useful equation for the gap temperature dependence
Gross Zeitschrift fur Physik B: Condensed Matter 82, 243

\[ \Delta(T) = \Delta(0) \tanh \left( \frac{\pi T_c}{\Delta(0)} \sqrt{\frac{1 - t}{t}} \right) \]

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</tbody>
</table>

node lines

isotropic gap

\[ \rho(t) = \frac{\Delta(0)}{\Delta(t)} \]
$d$-wave gap in cuprates

Linear T dependence due to quasi-particles ($e^-$) excited across the gap

From LNCMI Toulouse web page
Next lesson

Dirty superconductors

A phase diagram for superconductivity

Multi gap superconductivity

Commonalities
Flux pinning in YBa$_2$Cu$_3$O$_{6.95}$ crystals

(by Hardy Bonn and Liang)

Field cooling to 5.4 K in 0.5 T

Lowering the field at 5.4K by 11 mT
only muons in sample holder shift

The flux in the superconductor is trapped

Sonier Phys. Rev. Lett. 72 744