Introduction to $\mu$SR
(muon-spin rotation/relaxation)

Stephen J. Blundell

Clarendon Laboratory, Department of Physics, University of Oxford, UK

Muon training course - 2018
Introduction to $\mu$SR
(muon-spin rotation/relaxation)

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(see S.J. Blundell, Contemp. Phys. 40, 175 (1999) - also cond-mat/0207699)

Muon training course - 2018
Lecture plan

• Setting muons in their context, which in this case means a historical context!

• Muons were originally mis-identified. An important lesson in the history of science!

  “History always repeats itself; it has to, no-one listens.”

• Muons were first studied in cosmic rays; now produced in large accelerators
Muon spin rotation
Muon spin rotation
7 techniques using muons

TF-μSR

A_1

A_1^2

FT-μSR

ν →

ν →

LF-μSR

ZF-μSR

A_1

δA_1

ALC-μSR

B →

μSE

RF-μSR

A_1

ν →

t →

A_{RRF}
I.I. Rabi (1898-1988) - on the discovery of the muon:

“Who ordered that?”
Wilhelm Conrad ROENTGEN

Born 1845, Germany
Educated in Utrecht and Zurich
Died 1923
Nobel prize in 1901

discovered x-rays in 1895
Henri Becquerel (1852-1908)

1896 - discovered radioactivity

1903 Nobel prize
How to detect ionising radiation
More radiation here (300m from ground)

...than here

And also more than here (300m from tower in a horizontal direction)
Victor F. Hess (1883-1964)

Nobel prize 1936
(balloon flights 1911-1913)
Robert A. Millikan (1868-1953)

Oil drop experiment

first used the term “cosmic rays”

(music of the spheres)

founded Caltech
....but to really make progress, you need.....

... a research student!
Robert Millikan
- Nobel Prize 1923

his PhD student:
Carl D. Anderson
- Nobel Prize 1936
(shared with Hess)
Carl D. Anderson  
(1905-1991)  
discovered positron in 1931  
and in 1936 with Seth Neddermeyer discovered a particle called the **MESOTRON**  

mass $\sim 200 \ m_e$
Seth Neddermeyer  (one of Anderson’s grad students)
(and subsequently worked on implosions)
It is a dangerous thing...

...when experiments agree with theory
Yukawa (1907-1981):

\( p^+ \) and \( e^- \) interact by the exchange of virtual photons (QED)

⇒ if the strong force in the nucleus is mediated by exchange of virtual mesobrons, then need to “borrow” energy \( \Delta E \) given by

\[
\Delta E \Delta t \sim \hbar
\]

size of nucleus

\[
\frac{C}{2 \text{ fm}}
\]

\[\Delta E \sim \frac{10^{-34} \times 3 \times 10^8}{2 \times 10^{-15}} \]

\[\sim 100 \text{ MeV} \]

or \[200 \text{ me}^\circ\]

⇒ Anderson/Neddermeyer = Yukawa particle

Hideo Yukawa (1907-1981)
Nobel prize 1949
Bruno Rossi

lifetime of “mesotron” = 2.15(7) microseconds (1942)
Marcello Conversi

(1917-1988)
Muons STOP in a sample
- they do not diffract, or reflect
Conversi, Pancini, Piccioni experiments (1947)
Results of the experiment to implant cosmic ray muons in matter and measure their lifetime:

\[ \mu^+ \text{ in anything} \quad 2 \mu s \]
\[ \mu^- \text{ in C} \quad 2 \mu s \]
\[ \mu^- \text{ in Pb} \quad 0.07 \mu s \]

hence, mesotrons (muons) are not interacting strongly enough to be Yukawa’s particle!

Reason for \( \mu^- \) interaction: \( \mu^- \) capture:

\[ \mu^- + p^+ \rightarrow n + \nu_\mu \]

(radius of \( \mu^- \) orbit) \( \sim (m_e/m_\mu) \times \) (radius of e\(^-\) orbit)

so the \( \mu^- \) occupies an orbit close to the nucleus, and sees the full +Ze charge on nucleus.
Cecil Powell  (1903-1969)

1947 - discovered the pion

Bristol University

Nobel Prize 1950

pion mass  139.6 MeV
muon mass  105.7 MeV
Cosmic rays [mostly muons!]

Muons at ground level - average energy 4 GeV
For E > 1 GeV get ~ vertically 1 muon cm$^{-2}$ min$^{-1}$
And particle physics has kept on rolling....

...and many more!
### Particle properties

<table>
<thead>
<tr>
<th></th>
<th>charge</th>
<th>spin</th>
<th>mass</th>
<th>moment</th>
<th>$\frac{\gamma}{2\pi}$ (MHz T$^{-1}$)</th>
<th>lifetime ($\mu$s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>±e</td>
<td>$1/2$</td>
<td>$m_e$ = 0.51 MeV</td>
<td>657 $\mu_p$</td>
<td>28 x 10$^3$</td>
<td>$\infty$</td>
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<tr>
<td>$\mu$</td>
<td>±e</td>
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<td>3.18 $\mu_p$</td>
<td>135.5</td>
<td>2.19</td>
</tr>
<tr>
<td>$p$</td>
<td>±e</td>
<td>$1/2$</td>
<td>$1836 m_e$ = 938 MeV</td>
<td>$\mu_p$</td>
<td>42.6</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

* Muon beams have 100% spin polarization.
* Samples do not have to be deuterated.
**STEP 1:**

**Production of muons**

- Pions produced by \( p+p \rightarrow \pi^+ + p + n \)
- Pion decays in 26 ns
  \[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
- This is a **TWO-BODY** decay

\[ E_k > 280 \text{ MeV} \sim 2m_\pi c^2 \]
Pions that decay at rest on the surface of the target are called **surface muons**

If the pion is initially at rest:

(i) \[ E_\mu = \left[ \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \right] c^2 = 109.8 \text{ MeV} = m_\mu c^2 + 4.1 \text{ MeV} \]

(ii) \[ p_\mu = \left[ \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \right] c = 29.8 \text{ MeV}/c \]

(iii) \[ \beta = \frac{c p_\mu}{E_\mu} = 0.271 \]

(iv) Muon beam **100% spin-polarized**

Pions that decay in flight are called **decay muons** (they have less polarization, but higher energy [higher than 4.1 MeV] and so penetrate further into the sample).
Relativistic four-momentum:

\[ P = \left( \frac{E}{c}, \mathbf{p} \right) \]

\[ mc = \gamma m_0 c \quad m\nu = \gamma m_0 \nu \]

\[ P \cdot \nu = \frac{E^2}{c^2} - p^2 = m_0^2 c^2 \]

Application to pion decay

\[ P_{\pi} = P_{\mu} + P_{\nu} \]

\[ (m_\pi c, 0) = \left( \frac{E_{\mu}}{c}, p_{\mu} \right) + \left( -|p_{\mu}|, -p_{\mu} \right) \]

\[ E_{\mu} = m_\pi c^2 + p_{\mu} c \]

\[ E_{\mu}^2 = p_{\mu}^2 c^2 + m_\pi^2 c^4 \]

\[ = E_{\mu}^2 - 2E_{\mu} m_\pi c^2 + (m_\pi^2 + m_\mu^2) c^4 \]

\[ \Rightarrow E_{\mu} = \left( \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \right) c^2 \]

\[ \Rightarrow p_{\mu} = \frac{E_{\mu}}{c} - m_\pi c = \left( \frac{m_\mu^2 - m_\pi^2}{2m_\pi} \right) c \]

\[ \beta = \frac{v}{c} = \frac{p}{E/c} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} \]
**STEP 2:**

**Implantation**

Muons in matter

\[ 4\text{ MeV} \quad \mu^+ \quad v \sim c/4 \]

\[ \text{Sample} \]

- Ionization of atoms, electron scattering
  \[ (10^{-10}\text{s}) \]

- 2-3 keV

- Muonium formation - (10^{-12}\text{s})

- Successive $e^-$ capture & loss

- 200 eV
  \[ v \sim 0.002c \]

- Inelastic collisions between muonium and host atoms
  \[ (10^{-12}\text{s}) \]

- Few eV
STEP 3: decay

This is what every single muon experiment looks like!
Muons decay

- The muon decay occurs much more leisurely than the pion decay: half life 2.2 μs

\[
\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu
\]

- This is a THREE-BODY decay
Therefore the decay positrons can have a range of energies:

\[ E_{e^+} \approx 0 \]

\[ E_{e^+} \approx \frac{1}{2} m_{\mu} c^2 \]
Leon Lederman (1922–)
Nobel Prize 1988
Director of Fermilab 1979–1989
Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

RICHARD L. GARWIN,† LEON M. LEDERMAN, and MARCEL WEINRICH

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York
(Received January 15, 1957)

Fig. 1. Experimental arrangement. The magnetizing coil was close wound directly on the carbon to provide a uniform vertical field of 79 gauss per ampere.
“How we violated parity in a weekend .. and discovered God”
“I cannot believe God is a weak left-hander”

Wolfgang Pauli
(1900-1958)
Muon decay

Muon decays into a positron:

\[ \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \]
Muon decay

Muon decays into a positron:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

Positron decay is asymmetric with respect to the initial muon-spin polarization because of parity violation (weak interaction)
\[ a_0 = \frac{1}{3} \]
\[ \epsilon = \frac{E}{E_{\text{max}}} \]
\[ E_{\text{max}} = 52 \text{ MeV} \]
\[ a_0(\epsilon) = \frac{2\epsilon - 1}{3 - 2\epsilon} \]
\[ p(\theta) \propto 1 + a_0(\epsilon) \cos \theta \]
Various other materials were investigated for $\mu^+$ mesons. Nuclear emulsion as a target was found to have a significantly weaker asymmetry (peak-to-valley ratio of $1.40 \pm 0.07$) and it is interesting to note that this did not increase with reduced delay and gate width. Neither was there any evidence for an altered moment. It seems possible that polarized positive and negative muons will become a powerful tool for exploring magnetic fields in nuclei (even in Pb, 2% of the $\mu^-$ decay into electrons$^9$), atoms, and interatomic regions.
Spin-precession frequencies

\[ \mu = \gamma J \]

magnetic momentartery angular momentum

\[ \gamma = \frac{g \mu_N}{h} \]

gyromagnetic ratio
g-value

nuclear magneton

Classically:

\[ I = e \frac{v}{2\pi r} \]

\[ \Rightarrow \mu = \frac{I}{\pi r^2} = \frac{e}{2m_p} J \]
Nuclear magneton
\[ \mu_N = \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27} \text{ JT}^{-1} \]

Bohr magneton
\[ \mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ JT}^{-1} \]

<table>
<thead>
<tr>
<th></th>
<th>( g )</th>
<th>( J/h )</th>
<th>( \mu )</th>
<th>( \delta/2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROTON</td>
<td>5.5883</td>
<td>1/2</td>
<td>( 2.793\mu_N )</td>
<td>42.8 MHz T(^{-1})</td>
</tr>
<tr>
<td>ELECTRON</td>
<td>2.00</td>
<td>1/2</td>
<td>( -\mu_B )</td>
<td>28.1 GHz T(^{-1})</td>
</tr>
<tr>
<td>MUON</td>
<td>2.00</td>
<td>1/2</td>
<td>( 8.9\mu_N )</td>
<td>136 MHz T(^{-1})</td>
</tr>
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</table>
\[ \omega_e = \gamma_e B_e \]
\[ \omega_\mu = \gamma_\mu B_\mu \]
\[ \omega_p = \gamma_p B_p \]
Experiments at the ISIS muon facility
μSR and ordered organic ferromagnets and antiferromagnets
μSR and ordered organic ferromagnets and antiferromagnets

\[ A(t) \]

- **Ferromagnet**
- **Antiferromagnet**
µSR and ordered organic ferromagnets and antiferromagnets
Dipole fields

\[ B^\alpha(r_\mu) = \sum_i D^{\alpha \beta}_i (r_\mu) m_{i}^\beta \]

Dipole field at muon-site \( i \)th moment
Dipole fields

\[ B^{\alpha}(\mathbf{r}_\mu) = \sum_i D^{\alpha \beta}_i (\mathbf{r}_\mu) m^{\beta}_i \]

Dipole field at muon-site

\( i^{th} \) moment

\(|B|\)
### Summary

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<tr>
<th>Particle</th>
<th>Spin</th>
<th>Mass</th>
<th>Magnetic Moment</th>
<th>Gyromagnetic Ratio</th>
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