


WIR SCHAFFEN WISSEN – HEUTE FÜR MORGEN



Case study: elemental superconductors

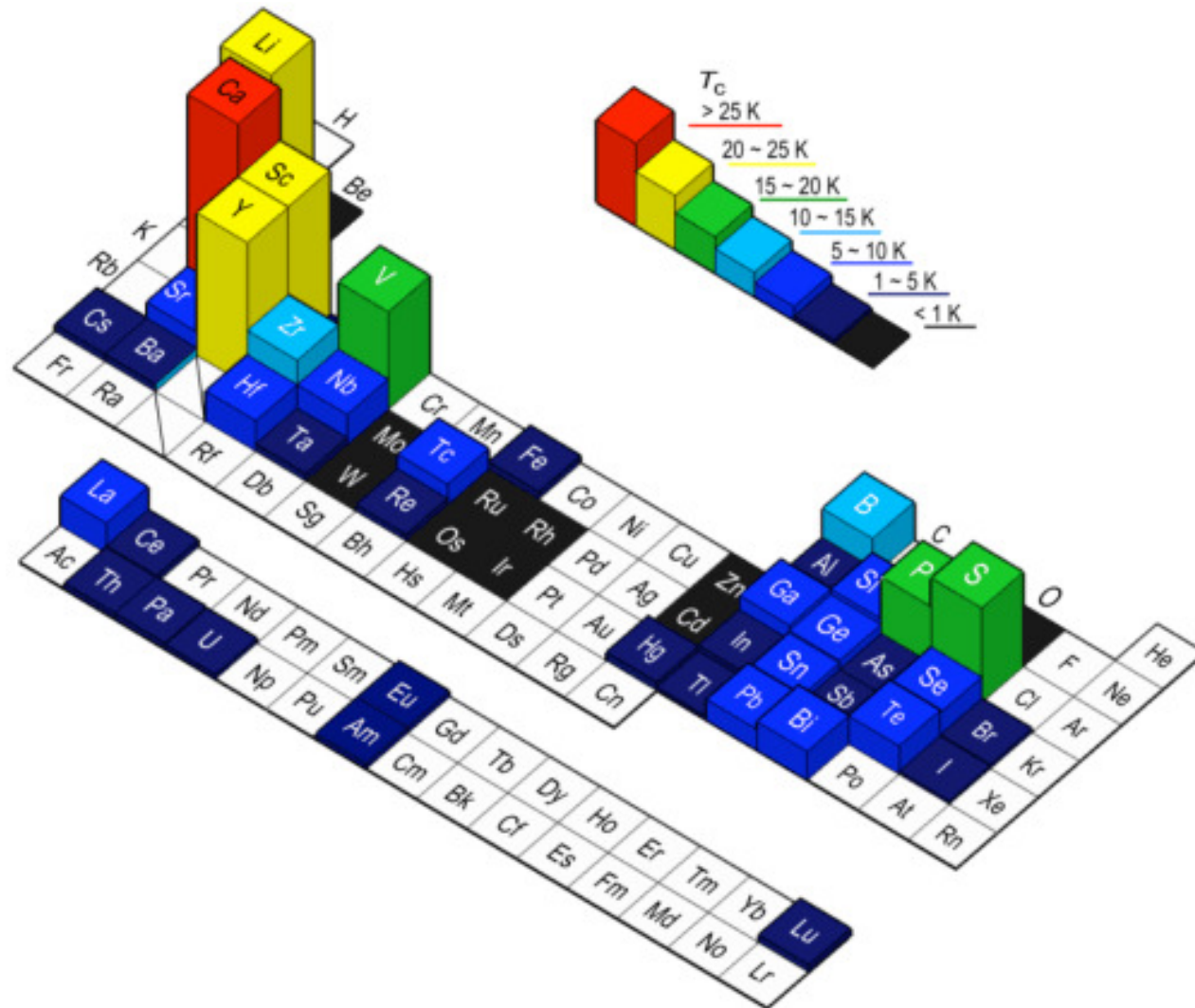
Rustem Khasanov, PSI, Switzerland

International Advanced School on Muon Spectroscopy, 15-22 August 2019

- 
- A solid grey square located on the left side of the slide.
- Elemental superconductors
 - μ SR in studies of type-I superconductivity
 - Sn
 - Bi-II
 - Ga-II
 - Universal relations for type-I superconductors



Elemental superconductors



Most of elemental superconductors are of a type-I
 For some elemental superconductors the type of superconductivity is still not known!

Superconductivity

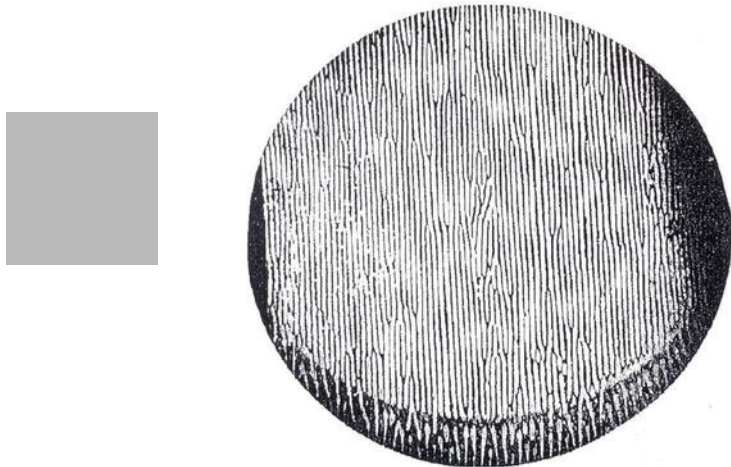
Superconducting elements

	T_c [K]		T_c [K]
Al	1.19	Ru	0.49
Cd	0.56	Sn	3.72
Ga	1.09	Ta	4.48
Hg	4.00	<u>Tc</u>	<u>8.22</u>
In	3.40	Th	1.37
Ir	0.14	Ti	0.39
La	5.00	Tl	2.39
Mo	0.92	U	0.68
<u>Nb</u>	<u>9.13</u>	V	5.30
Os	0.65	Zn	0.87
<u>Pb</u>	<u>7.19</u>	Zr	0.55
Re	1.70		

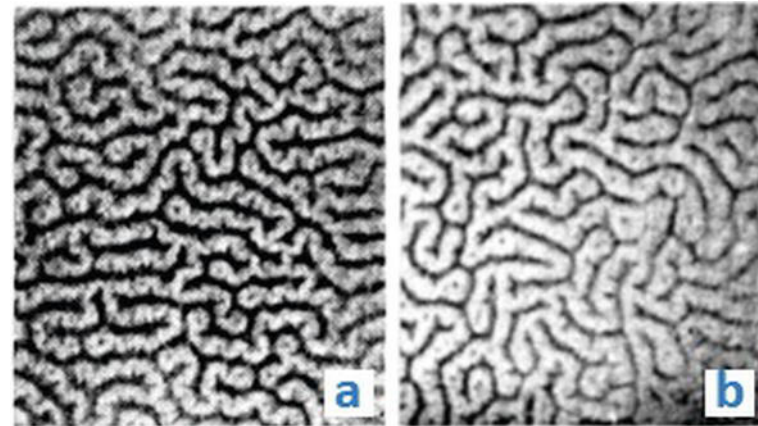


Intermediate state of type-I superconductors

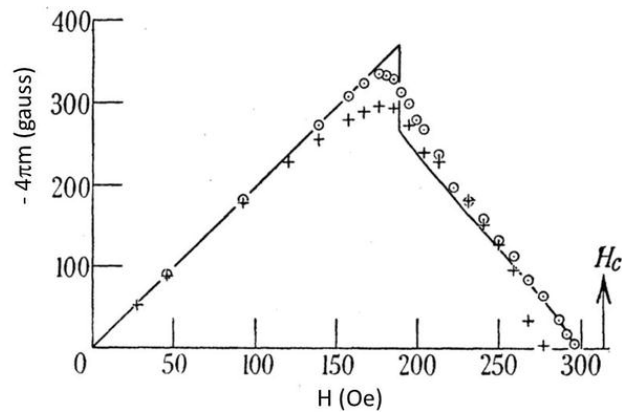
Intermediate state of type-I superconductor



Sharvin et al., ZhETPh, 33, 1341, 1957



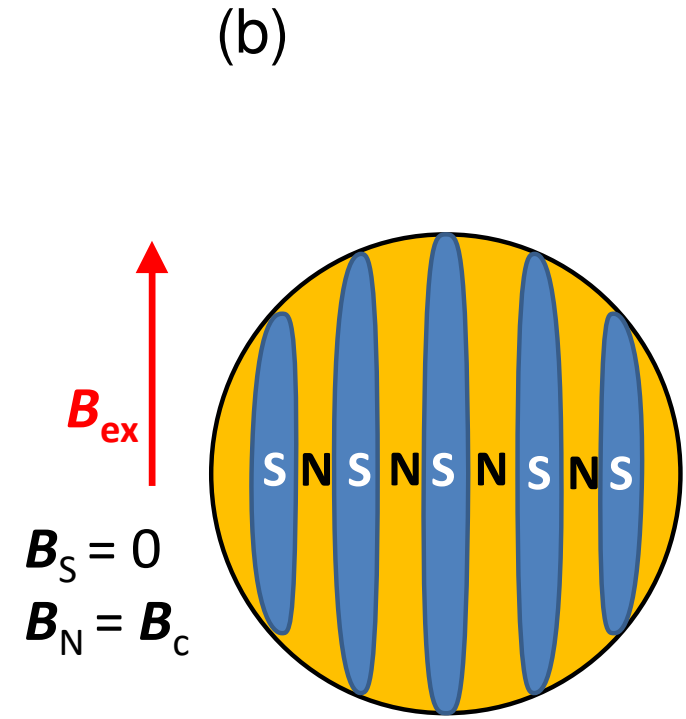
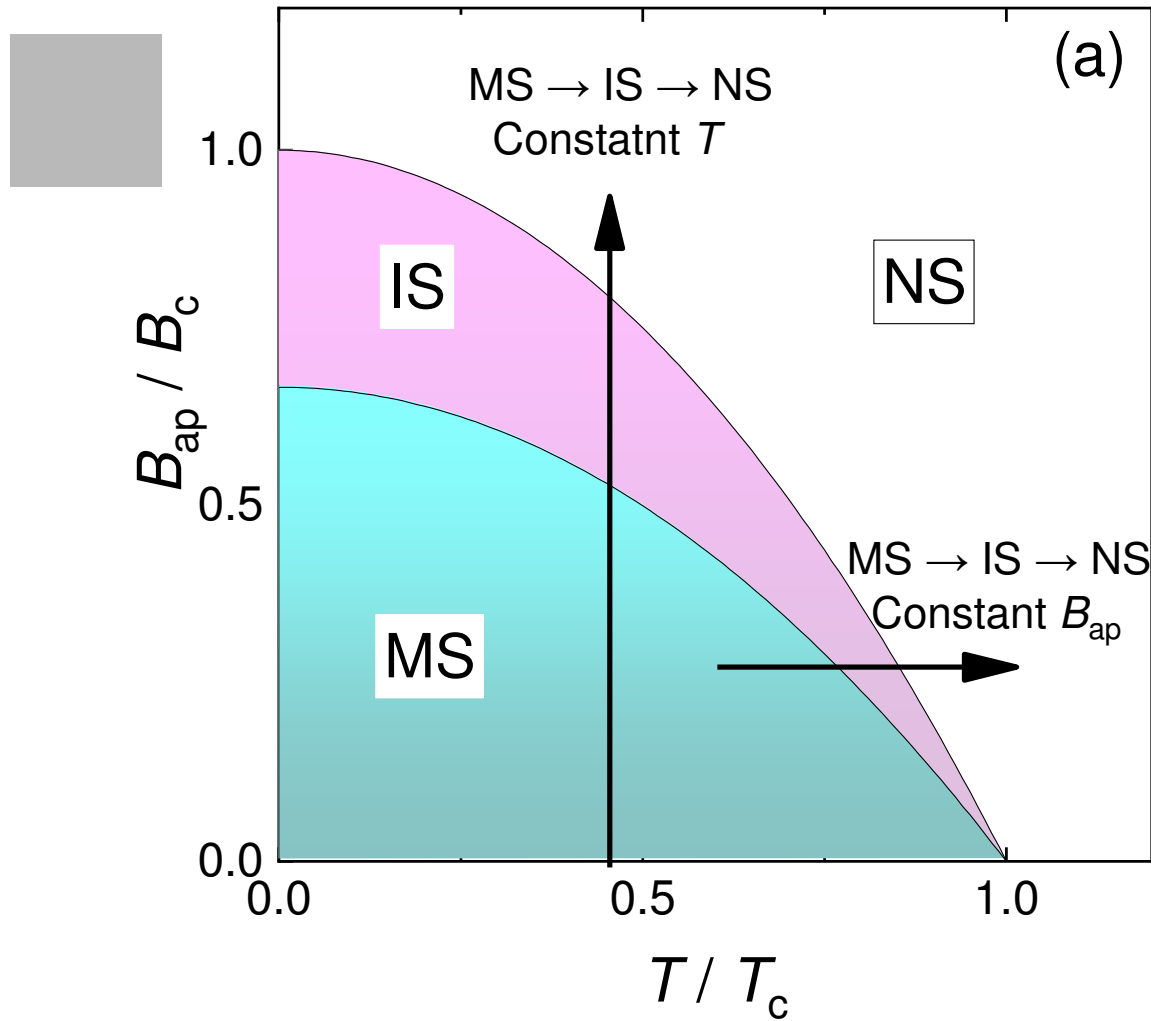
Faber, Proc.Roy.Soc., A248, 460 (1958)



Magnetization curve of cylindrical mercury specimen measured at increasing (\odot) and decreasing (\oplus) fields. H_c is thermodynamic critical field measured in parallel field.

Desirant and Shoenberg, Proc. Roy. Soc. 194, 63 (1948)

Intermediate state of the sphere

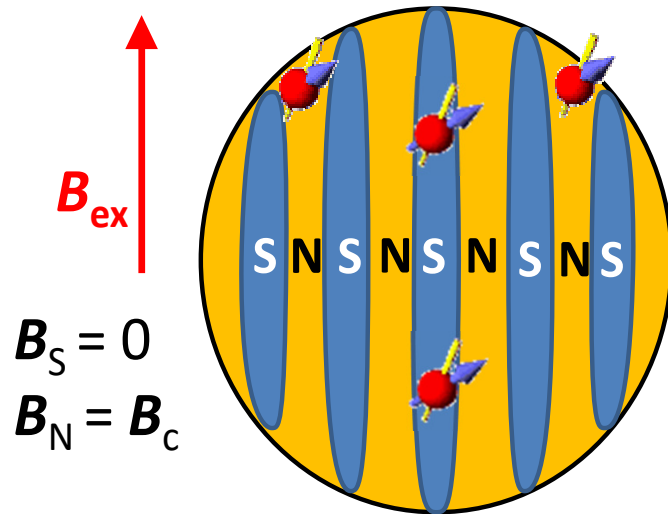




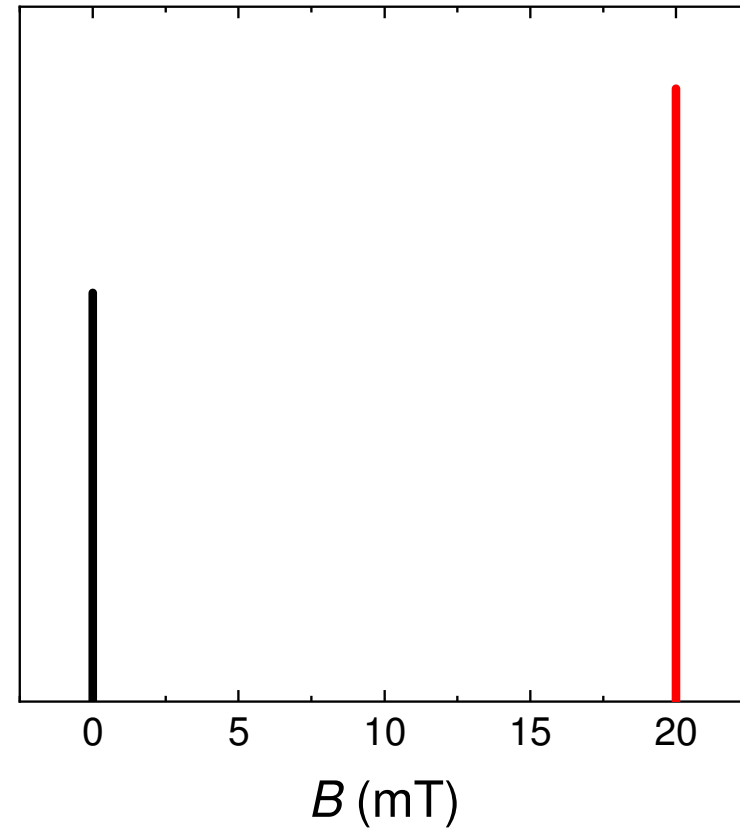
μ SR on type-I superconductors

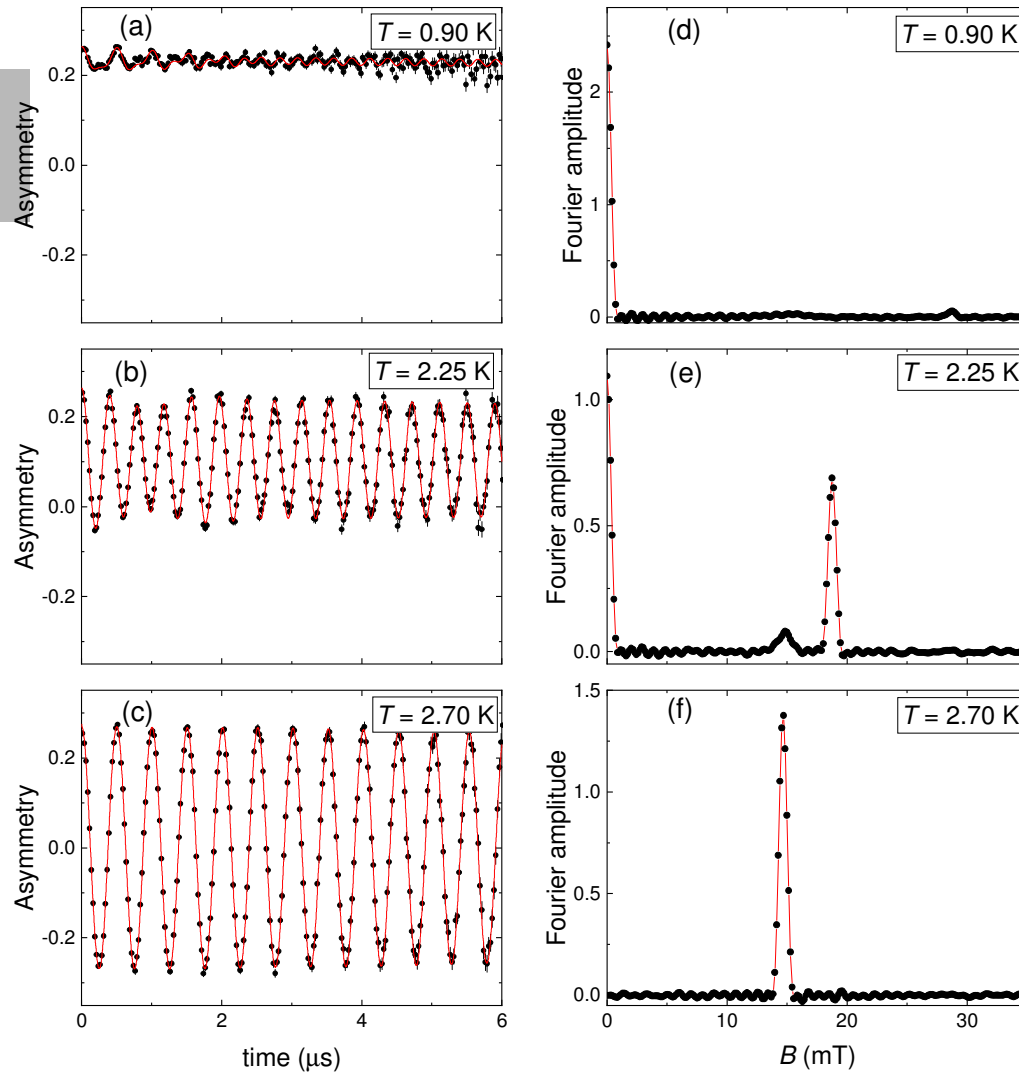
1. M. Gladisch, D. Herlach, H. Metz, H. Orth, G. zu Putlitz, A. Seeger, H. Teichler, W. Wahl, and W. Wigand, *Hyperfine Interact.* **6**, 109 (1979).
2. V. G. Grebinnik, I. I. Gurevich, V. A. Zhukov, A. I. Klimov, L. A. Levina, V. N. Maiorov, A. P. Manych, E. V. Mel'nikov, B. A. Nikol'skii, A. V. Pirogov, A. N. Ponomarev, V. S. Roganov, V. I. Selivanov, and V. A. Suetin, *Sov. Phys. JETP* **52**, 261 (1980).
3. V. S. Egorov, G. Solt, C. Baines, D. Herlach, and U. Zimmermann, *Phys. Rev. B* **64**, 024524 (2001).
4. V. S. Egorov, G. Solt, C. Baines, D. Herlach, and U. Zimmermann, *Physica B* **289-290**, 393 (2000).
5. V. Kozhevnikov, A. Suter, T. Prokscha, and C. Van Haesendonck, arXiv:1802.08299.
6. D. Singh, A. D. Hillier, and R. P. Singh, *Phys. Rev. B* **99**, 134509 (2019).
7. J. Beare, M. Nugent, M. N. Wilson, Y. Cai, T. J. S. Munsie, A. Amon, A. Leithe-Jasper, Z. Gong, S. L. Guo, Z. Guguchia, Y. Grin, Y. J. Uemura, E. Svanidze, and G. M. Luke, *Phys. Rev. B* **99**, 134510 (2019).
8. R. Khasanov, M. M. Radonjić, H. Luetkens, E. Morenzoni, G. Simutis, S. Schönecker, W. H. Appelt, A. Östlin, L. Chioncel, and A. Amato, *Phys. Rev. B* **99**, 174506 (2019).
9. R. Karl, F. Burri, A. Amato, M. Donegà, S. Gvasaliya, H. Luetkens, E. Morenzoni, and R. Khasanov, *Phys. Rev. B* **99**, 184515 (2019).
10. R. Khasanov, H. Luetkens, and E. Morenzoni, arxiv:1906.12253.

μ SR in type-I superconductors



$P(B)$ (arb. units)



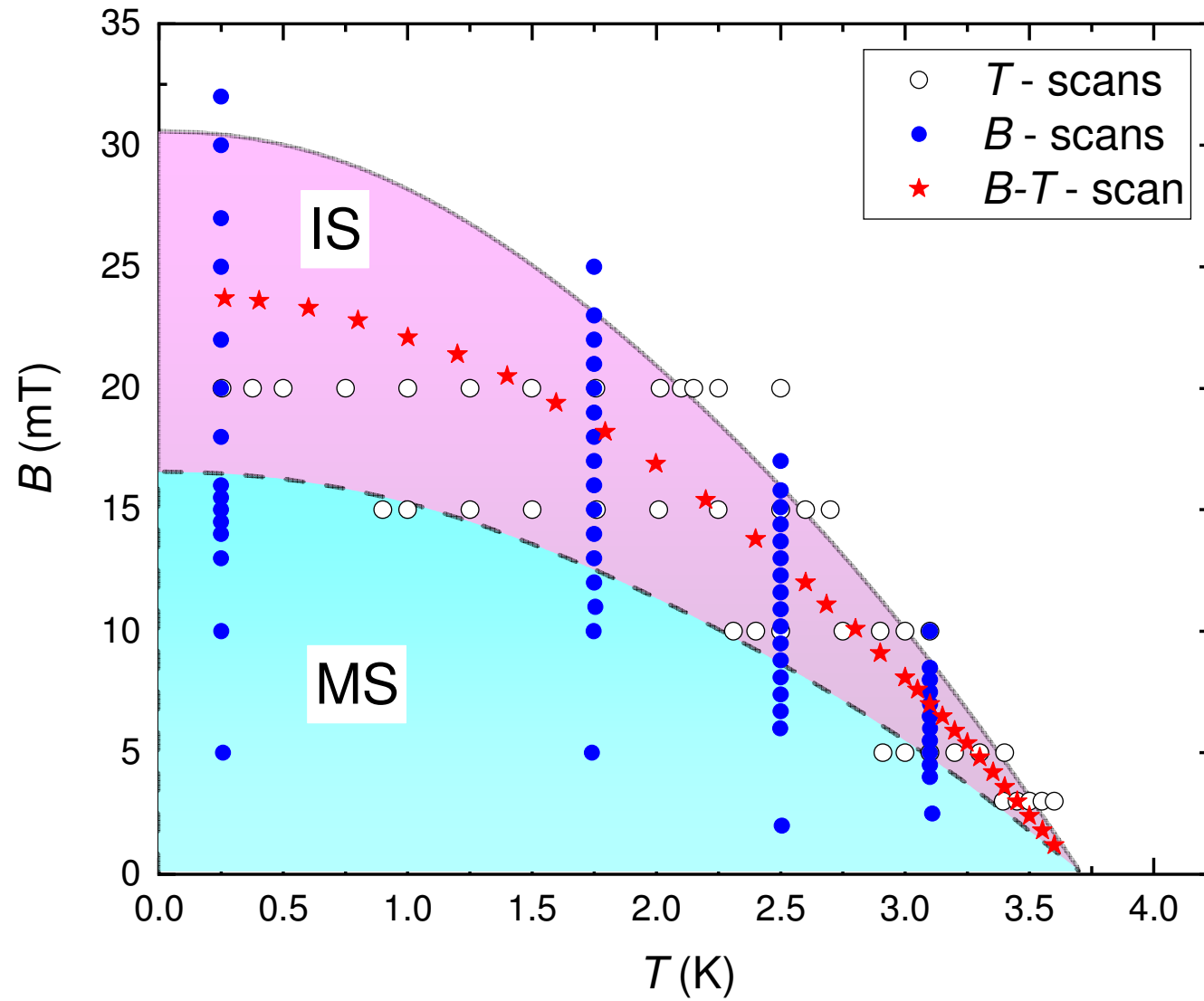


Raw data taken at $B_{\text{ap}} = 15\text{ mT}$ at $T = 0.90\text{ K}$ [panels (a) and (d)], 2.25 K [panels (b) and (e)], and 2.70 K [panels (c) and (f)].

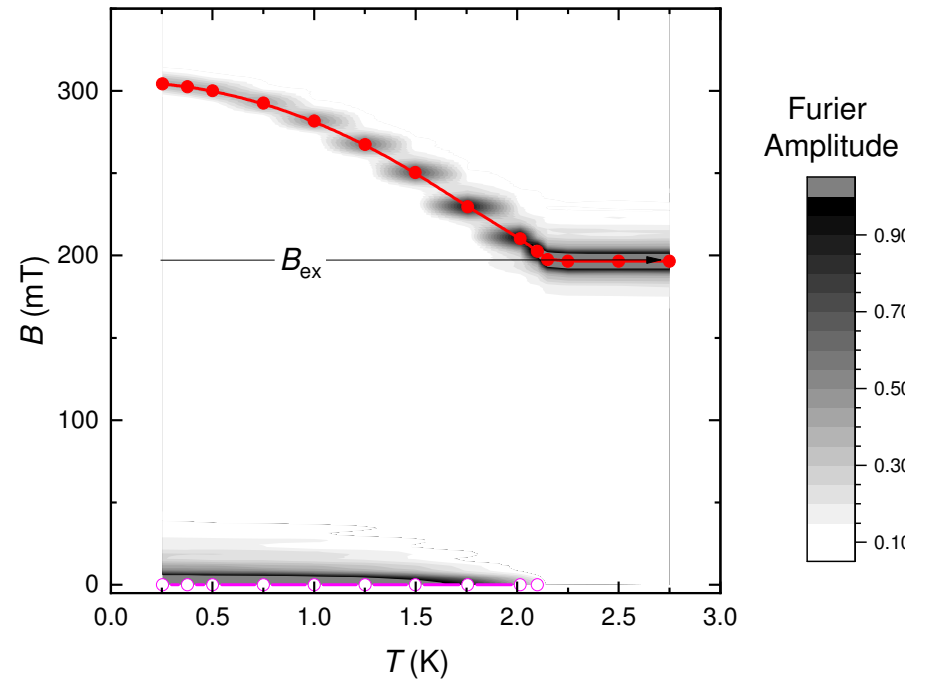
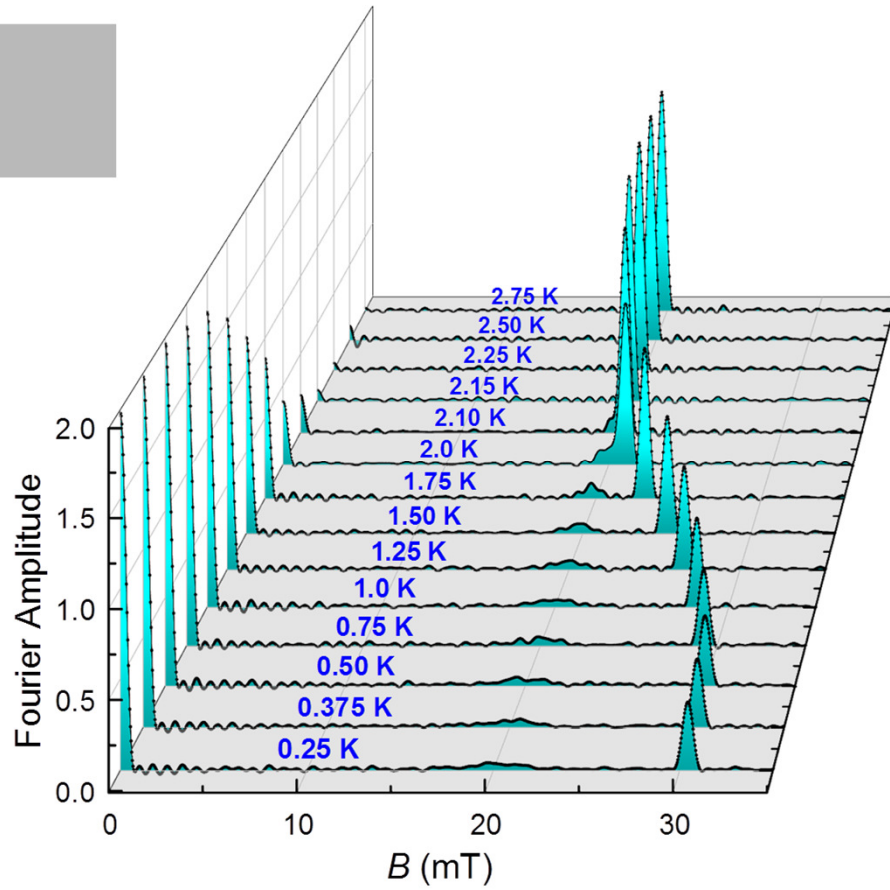


μ SR on β -Sn

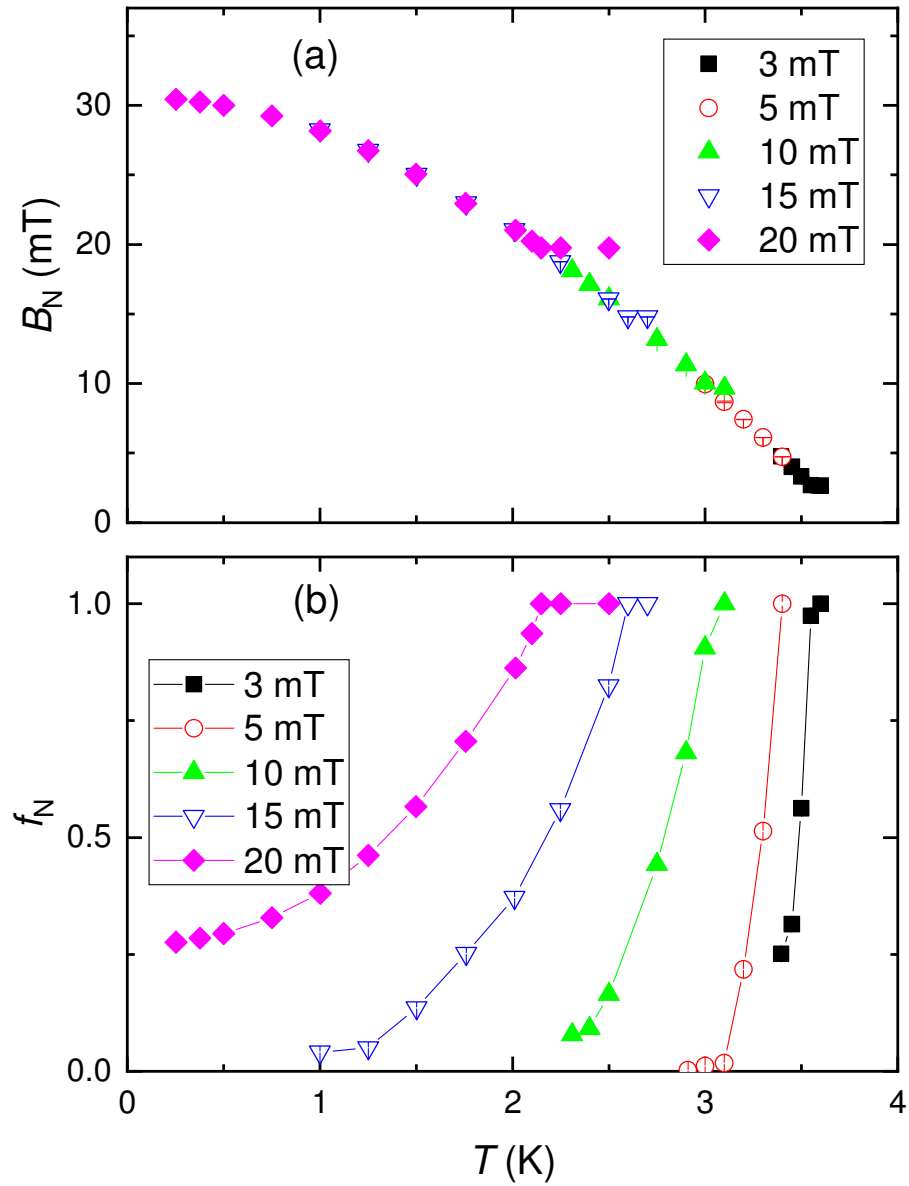
Way to measure: elemental Sn



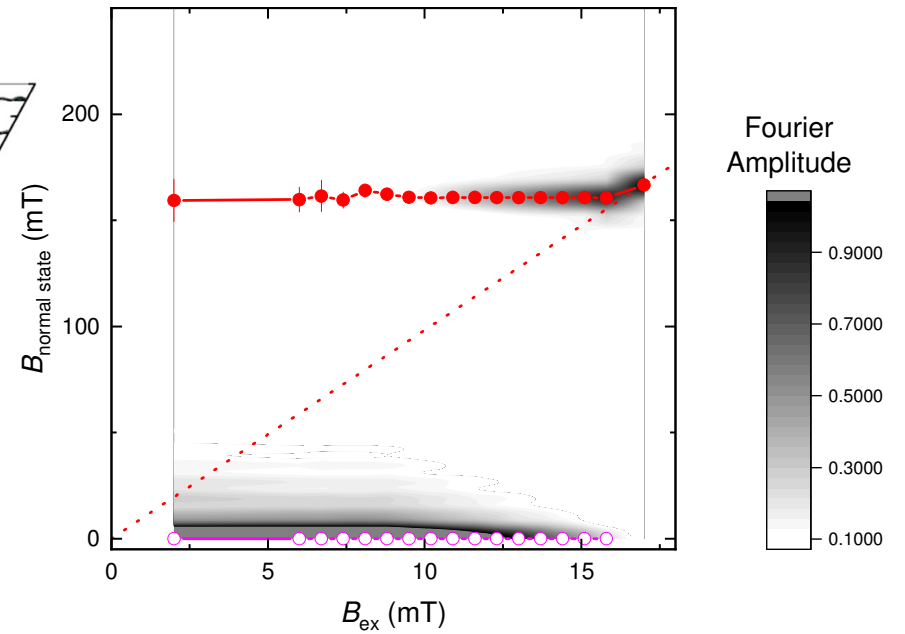
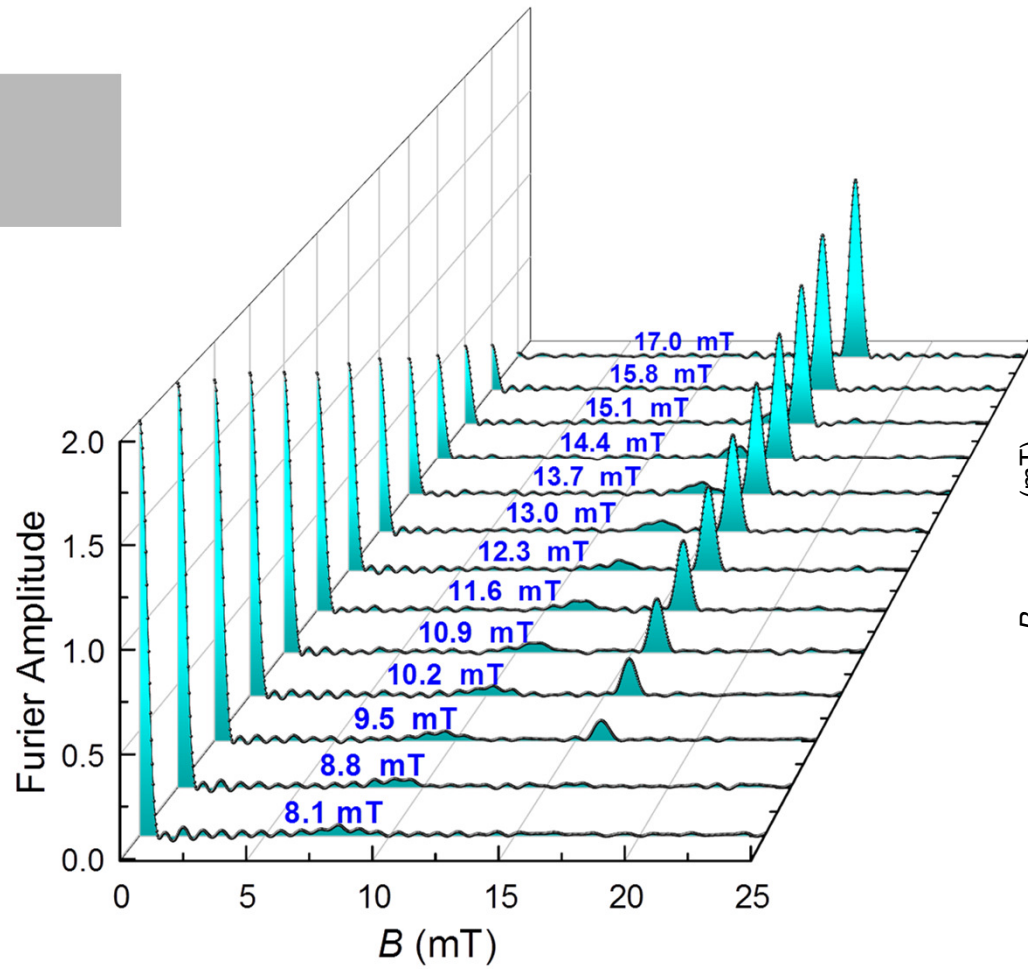
Temperature scans



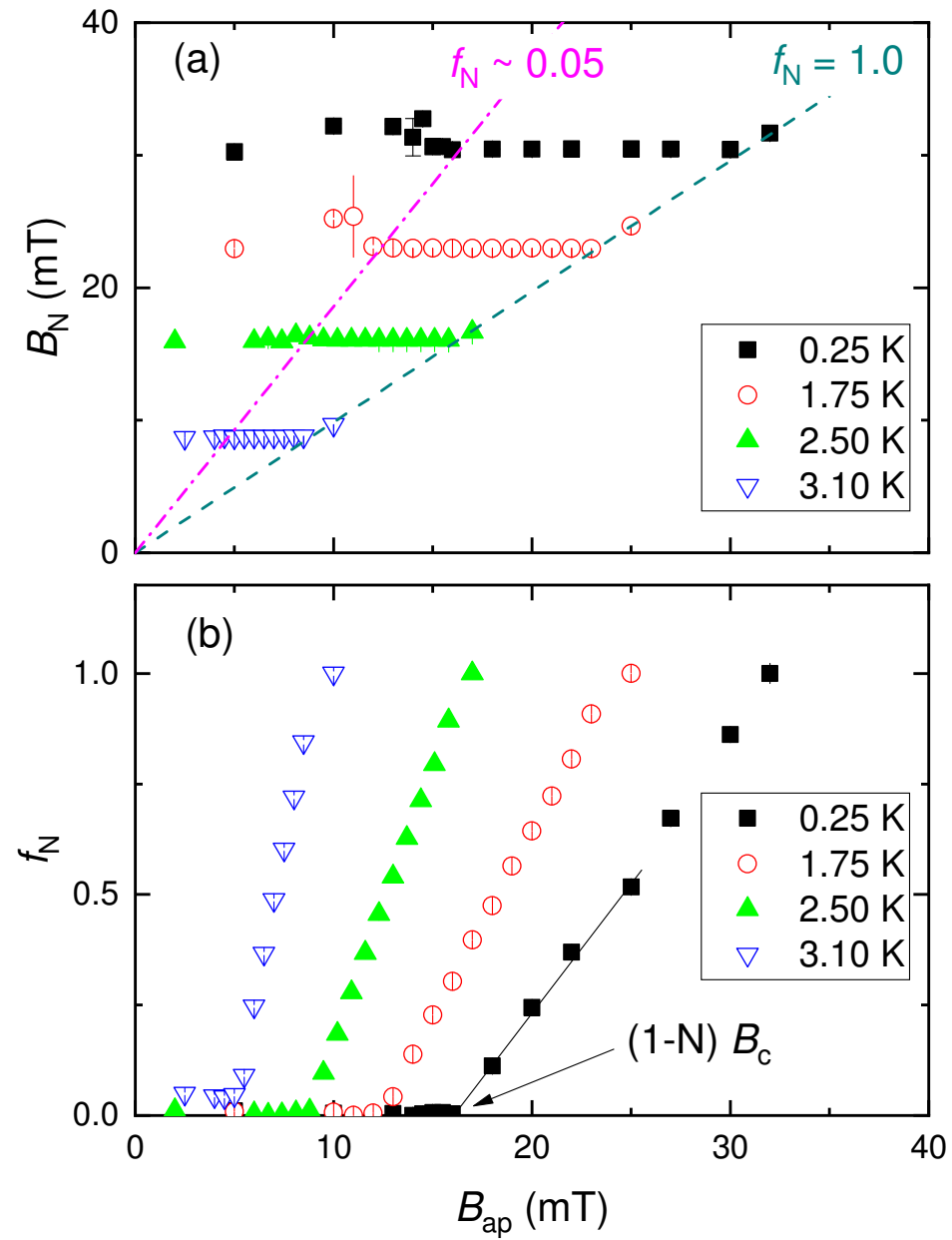
Temperature scans



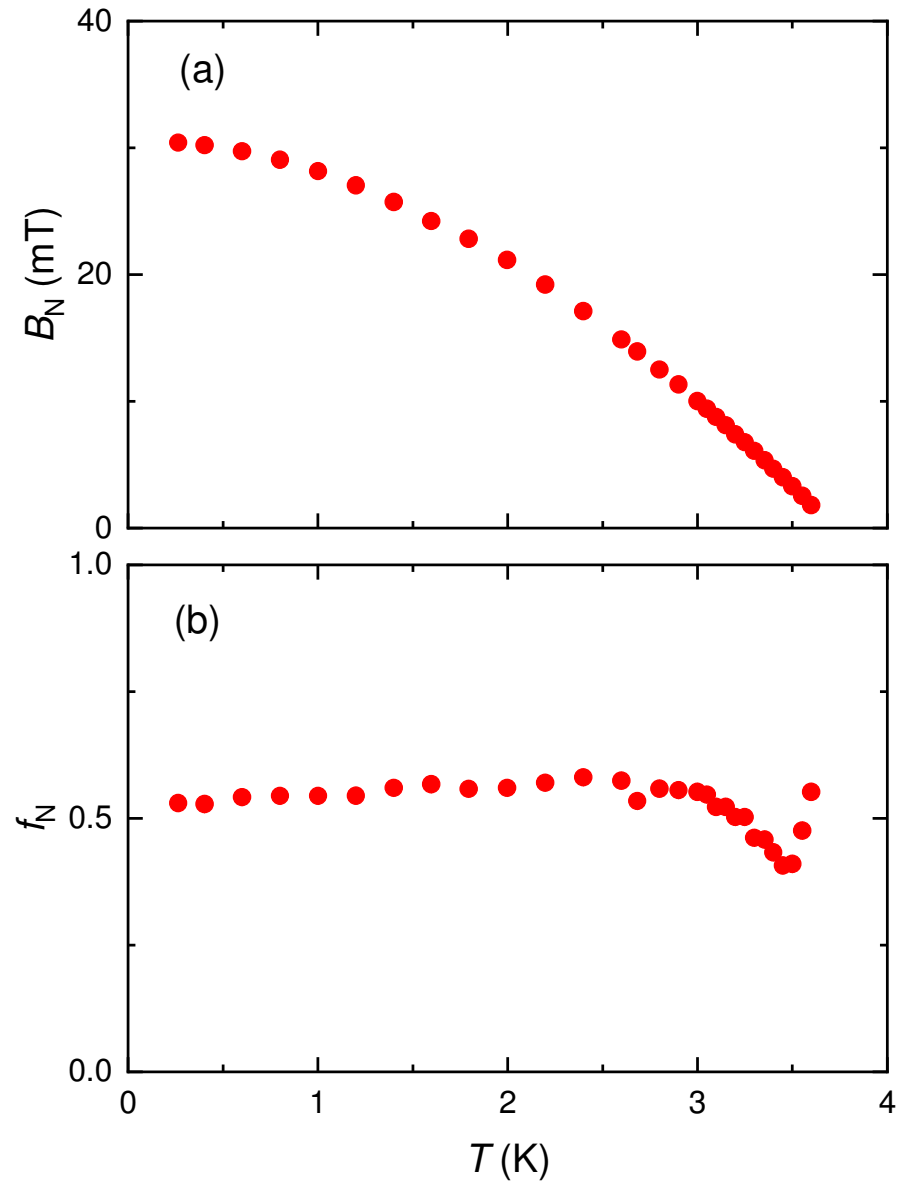
Field scan at $T=2.50$ K



Field scans



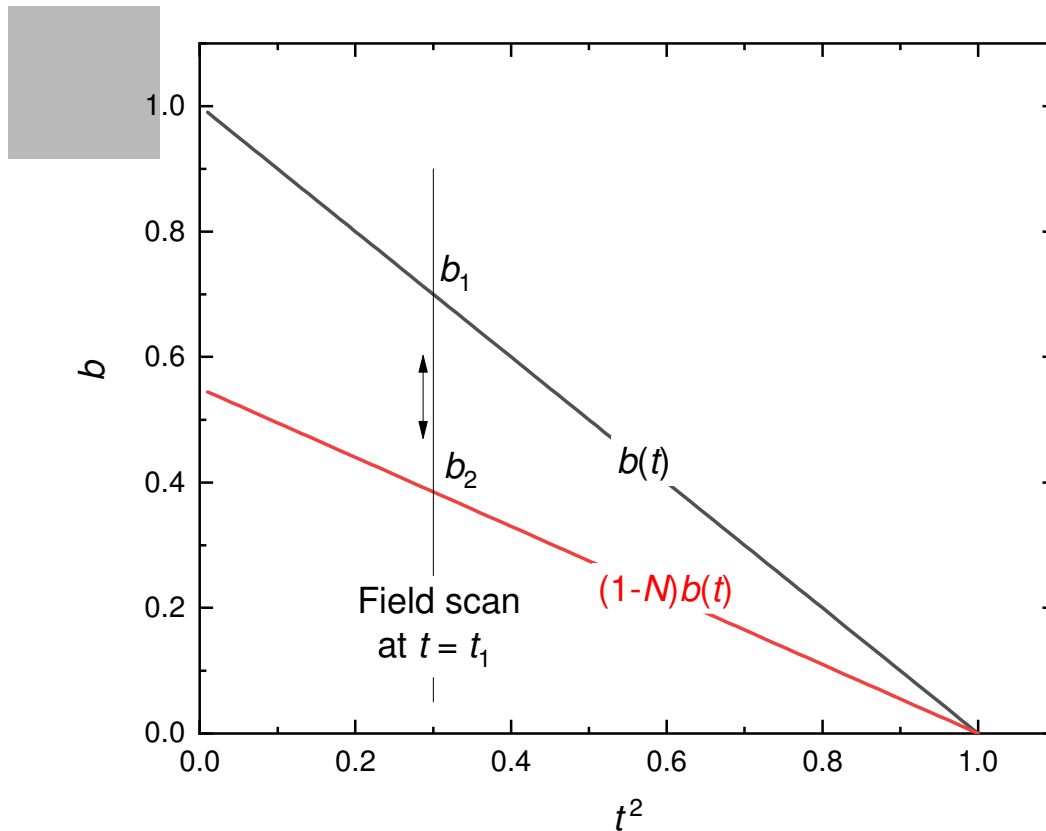
B-T scans





Information obtained from μ SR on type-I sc's

Demagnetization: scaling of $f(T)$ and $f(B)$



B-scans

$$f(b_1, t_1) = 1$$

$$f(b_2, t_1) = 0$$

$$b_2 = (1 - N)b_1$$



$$f(b, t_1) = \frac{b}{b_1} \frac{1}{N} - \frac{1 - N}{N}$$



$f(B)$ curves should scale with
 $1/B_c$ at corresponding T

Demagnetization: scaling of $f(T)$ and $f(B)$

T-scans

$$f(T) = \frac{T^2 - T_c^2}{T_c(B)^2 - T_c^2} \frac{N - 1}{N} + \frac{1}{N}$$



$f(T)$ curves scale with
 $1/(1-t_c^2)$ at corresponding B

B-scans

$$\begin{aligned} f(b_1, t_1) &= 1 \\ f(b_2, t_1) &= 0 \\ b_2 &= (1 - N)b_1 \end{aligned}$$



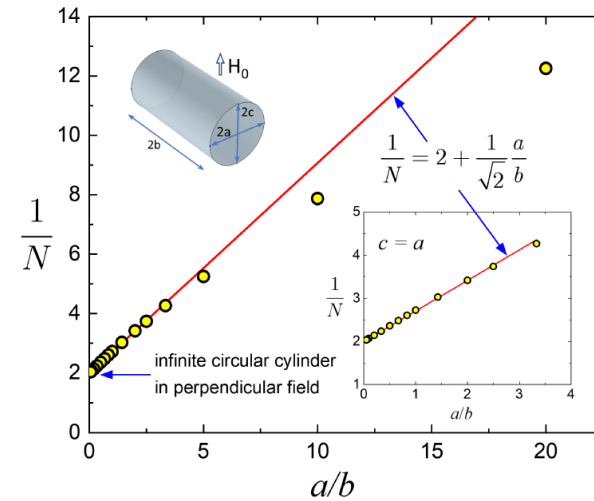
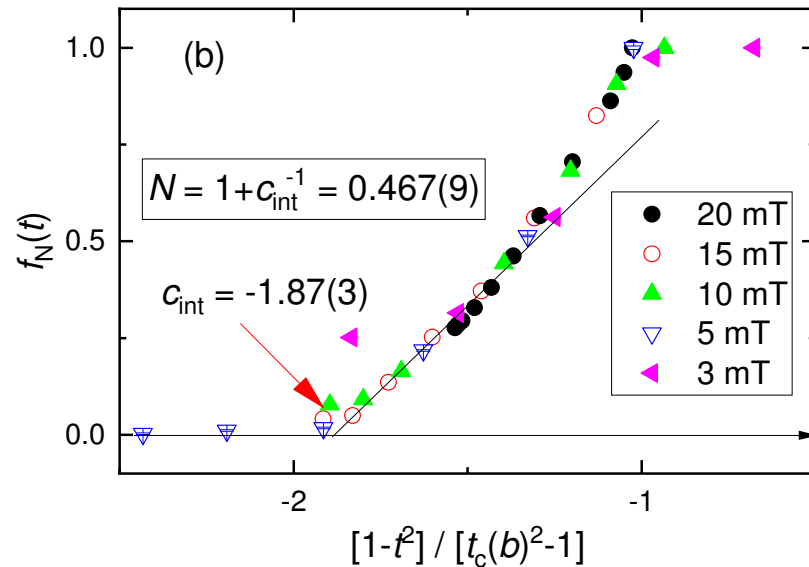
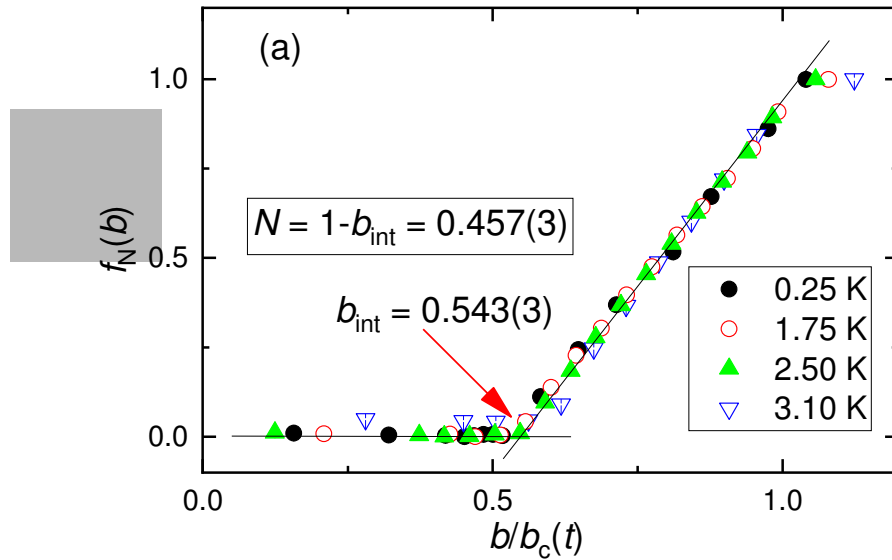
$$f(b, t_1) = \frac{b}{b_1} \frac{1}{N} - \frac{1 - N}{N}$$



$$f(B) = \frac{B}{B_c(T)} \frac{1}{N} - \frac{1 - N}{N}$$

$f(B)$ curves should scale with
 $1/B_c$ at corresponding T

Demagnetization: scaling of $f(T)$ and $f(B)$



Prozorov and Kogan, Phys. Rev. Applied **10**, 014030 (2018)

Sample dimensions: $\varnothing = 20.0$ mm, $h = 100$ mm

$$N_{\text{theor}} = 0.467. N_{\text{exp}} = 0.457 - 0.467$$

The single-band α -model of superconductivity was first developed by Padamsee *et al* [J. Low Temp. Phys. **12** 387 (1973)]. It is adapted from the single-band Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity to allow fits to electronic heat capacity versus T data that deviate from the BCS prediction. The model was further reconsidered by Johnston [*SST* **26**, 115001 (2013)].



The model assumes that the normalized superconducting order parameter $\Delta(T)/\Delta(0)$ is the same as in BCS theory, calculated using the weak-coupled BCS value $\alpha_{\text{BCS}} = \Delta(0) / k_{\text{B}} T_{\text{c}} = 1.764$. On the other hand, to calculate the electronic free energy, entropy, heat capacity and thermodynamic critical field versus T , the α -model takes α to be an adjustable parameter.

α -model. Gap equations

$$\int_0^{k_B \Theta_D} \frac{d\epsilon}{E} \tanh\left(\frac{E}{2k_B T}\right) = \frac{1}{N(0)V},$$

where

$$E = \sqrt{\epsilon^2 + \Delta^2}$$

$$\int_0^{k_B \Theta_D} \frac{d\epsilon}{E} \tanh\left(\frac{E}{2k_B T}\right) = \ln\left(\frac{2\Theta_D}{\alpha_{\text{BCS}} T_c}\right).$$

$$\int_0^{k_B \Theta_D} \frac{d\epsilon}{E} \tanh\left(\frac{E}{2k_B T}\right) = \ln\left[\frac{2k_B \Theta_D}{\Delta(0)}\right].$$

$$\int_0^{\frac{k_B \Theta_D}{\tilde{\Delta}(0)}} \frac{d\tilde{\epsilon}}{\tilde{E}} \tanh\left(\frac{\alpha_{\text{BCS}} \tilde{E}}{2t}\right) = \ln\left[\frac{2k_B \Theta_D}{\Delta(0)}\right],$$

$$\tilde{\Delta} = \frac{\Delta}{\Delta(0)}, \quad \tilde{\epsilon} = \frac{\epsilon}{\Delta(0)}, \quad t = \frac{T}{T_c}.$$

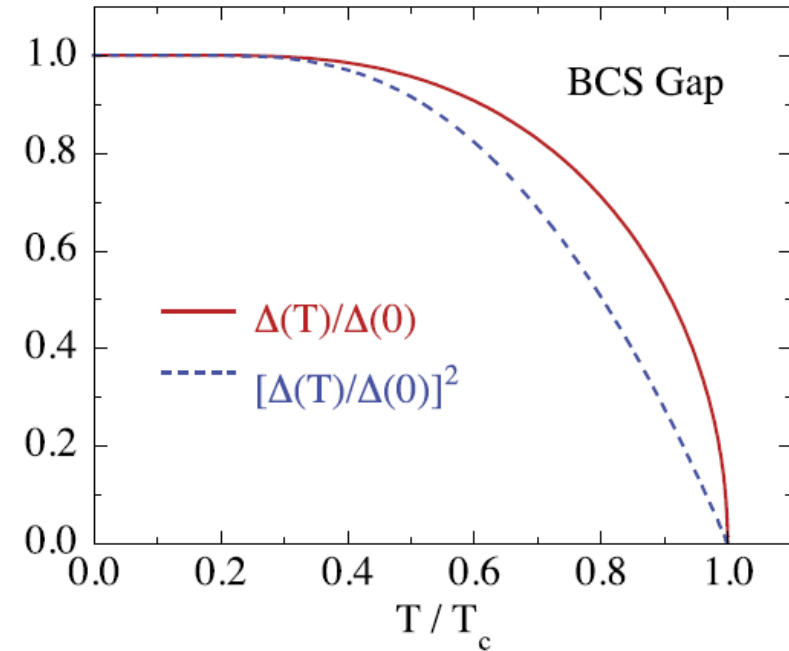


Figure 1. Normalized BCS energy gap $\Delta(T)/\Delta(0)$ and its square versus reduced temperature T/T_c as obtained by numerically solving equation (11) in the weak-coupling limit (1).

$$\frac{S_{es}(t)}{\gamma_n T_c} = \frac{6\alpha_{BCS}^2}{\pi^2 t} \int_0^\infty f(\alpha_{BCS}, \tilde{E}, t) \left(\tilde{E} + \frac{\tilde{\epsilon}^2}{\tilde{E}} \right) d\tilde{\epsilon},$$

$$\frac{C_{es}(t)}{\gamma_n T_c} = \frac{6\alpha_{BCS}^3}{\pi^2 t} \int_0^\infty f(1-f) \left(\frac{\tilde{E}^2}{t} - \frac{1}{2} \frac{d\tilde{\Delta}^2}{dt} \right) d\tilde{\epsilon},$$

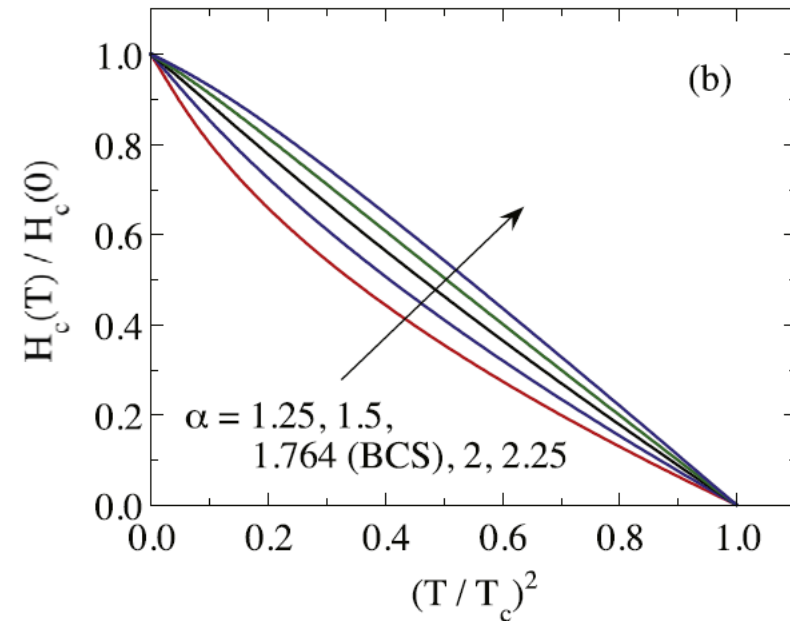
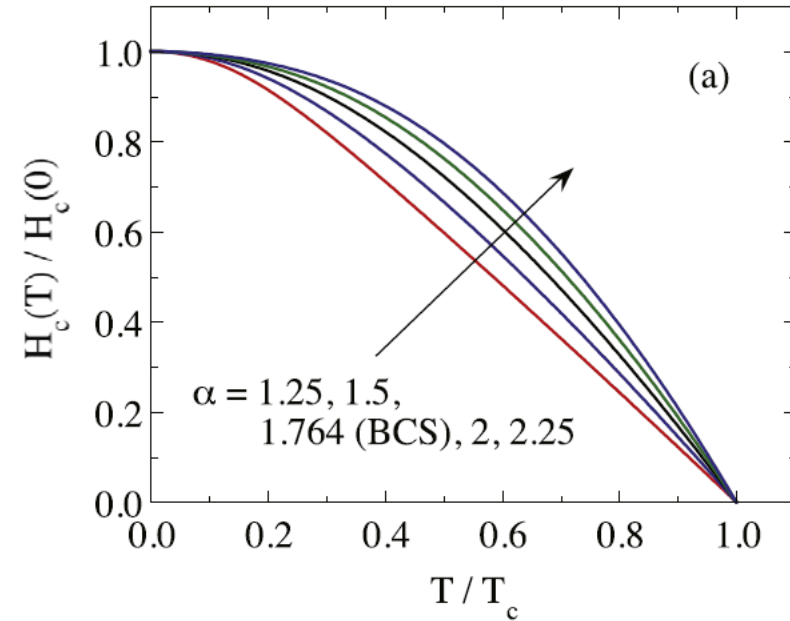
$$\frac{F_{es}(t)}{\gamma_n T_c^2} = -\frac{3\alpha_{BCS}^2}{4\pi^2} \times \left[\tilde{\Delta}^2 + 4 \int_0^\infty f(\alpha_{BCS}, \tilde{E}, t) \left(\frac{2\tilde{\epsilon}^2 + \tilde{\Delta}^2}{\tilde{E}} \right) d\tilde{\epsilon} \right].$$

$$f \equiv f(\alpha_{BCS}, \tilde{E}, t) = \frac{1}{e^{\alpha_{BCS} \tilde{E}/t} + 1}$$

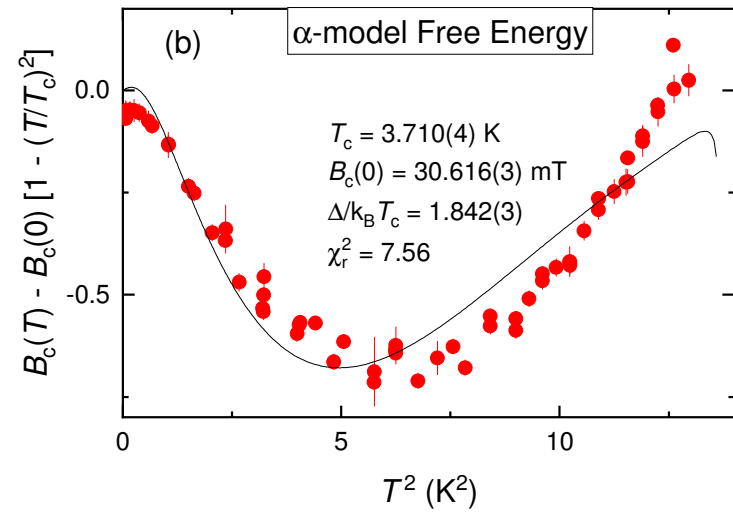
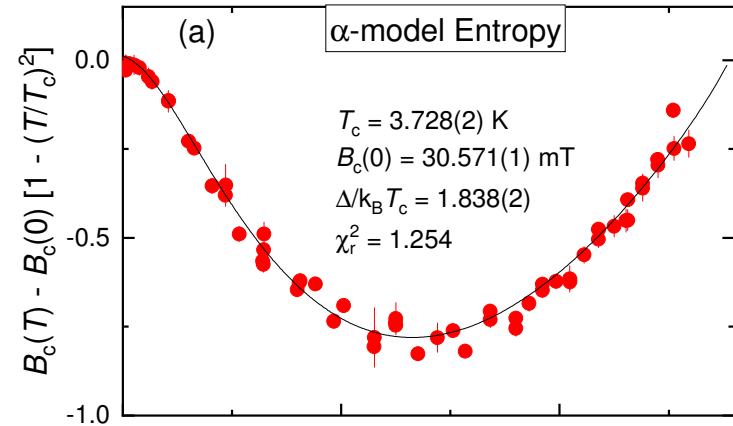
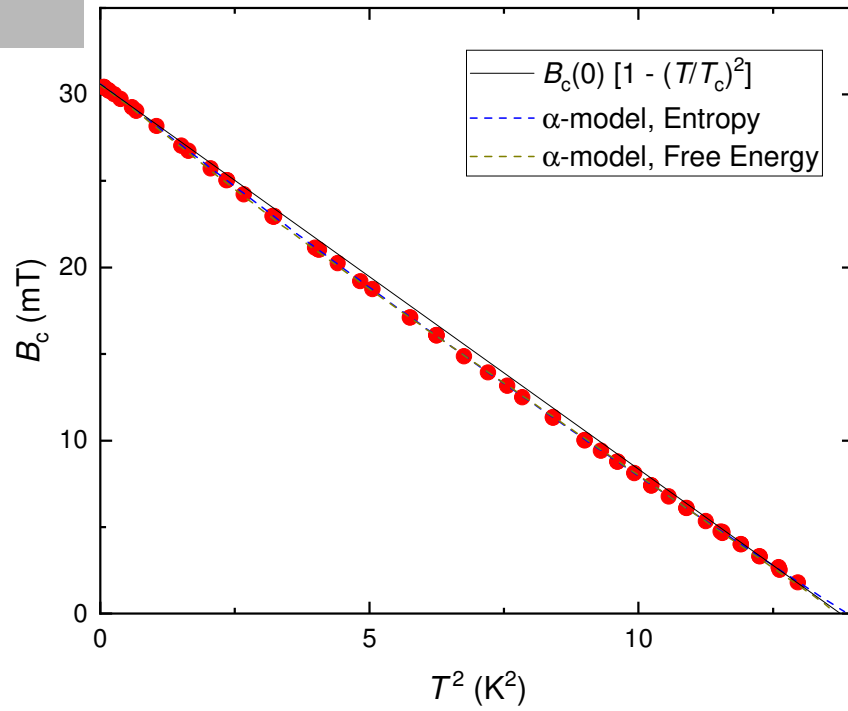
$$\text{with } \frac{E}{k_B T} = \frac{\alpha_{BCS} \tilde{E}}{t}.$$

$$\frac{B_c^2(t)}{8\pi} = F_N(t) - F_S(t),$$

$$\frac{B_c^2(t)}{8\pi} = T_c \int_t^1 [S_N(t') - S_S(t')] dt'$$



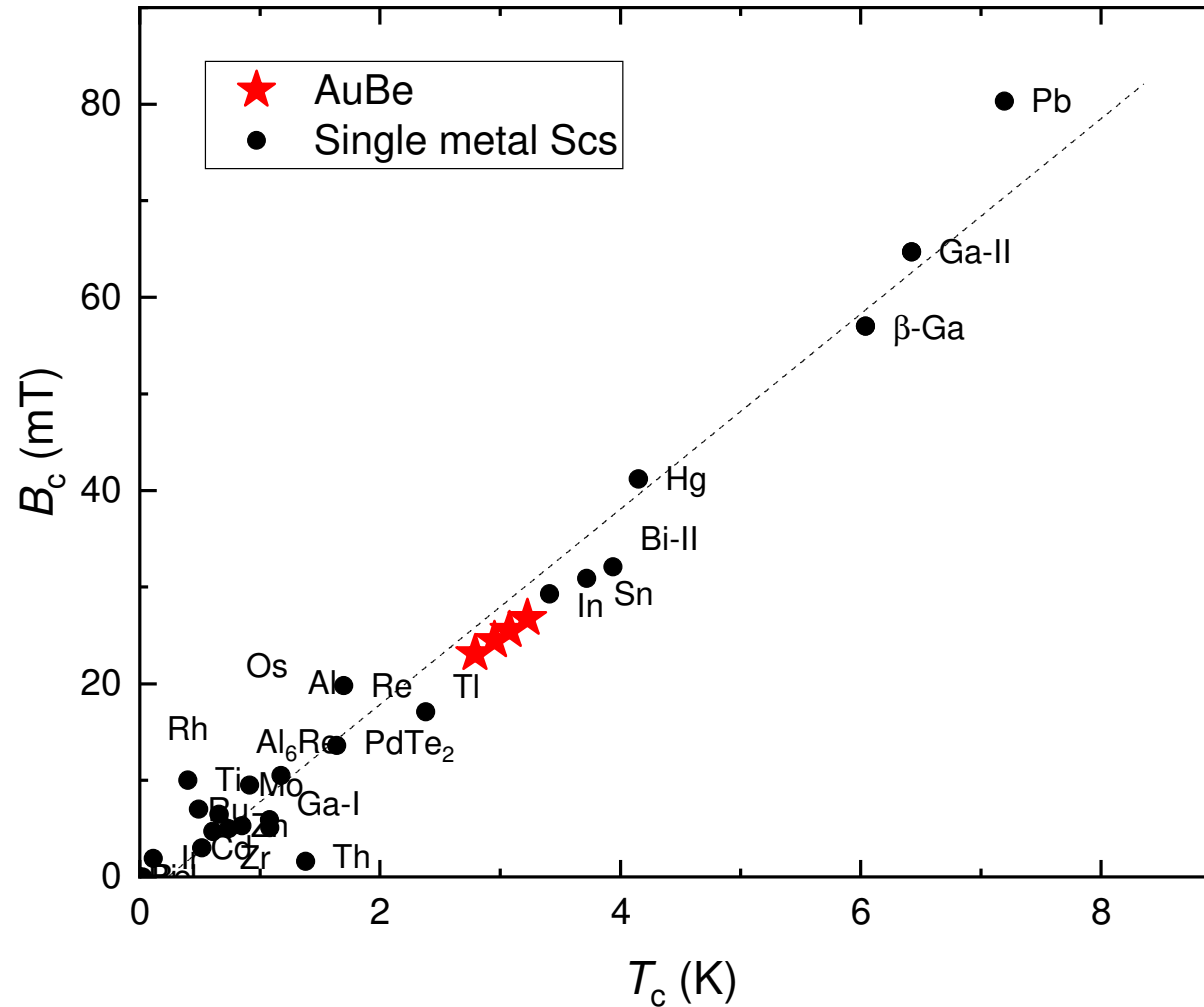
T-dependence of the thermodynamic critical field





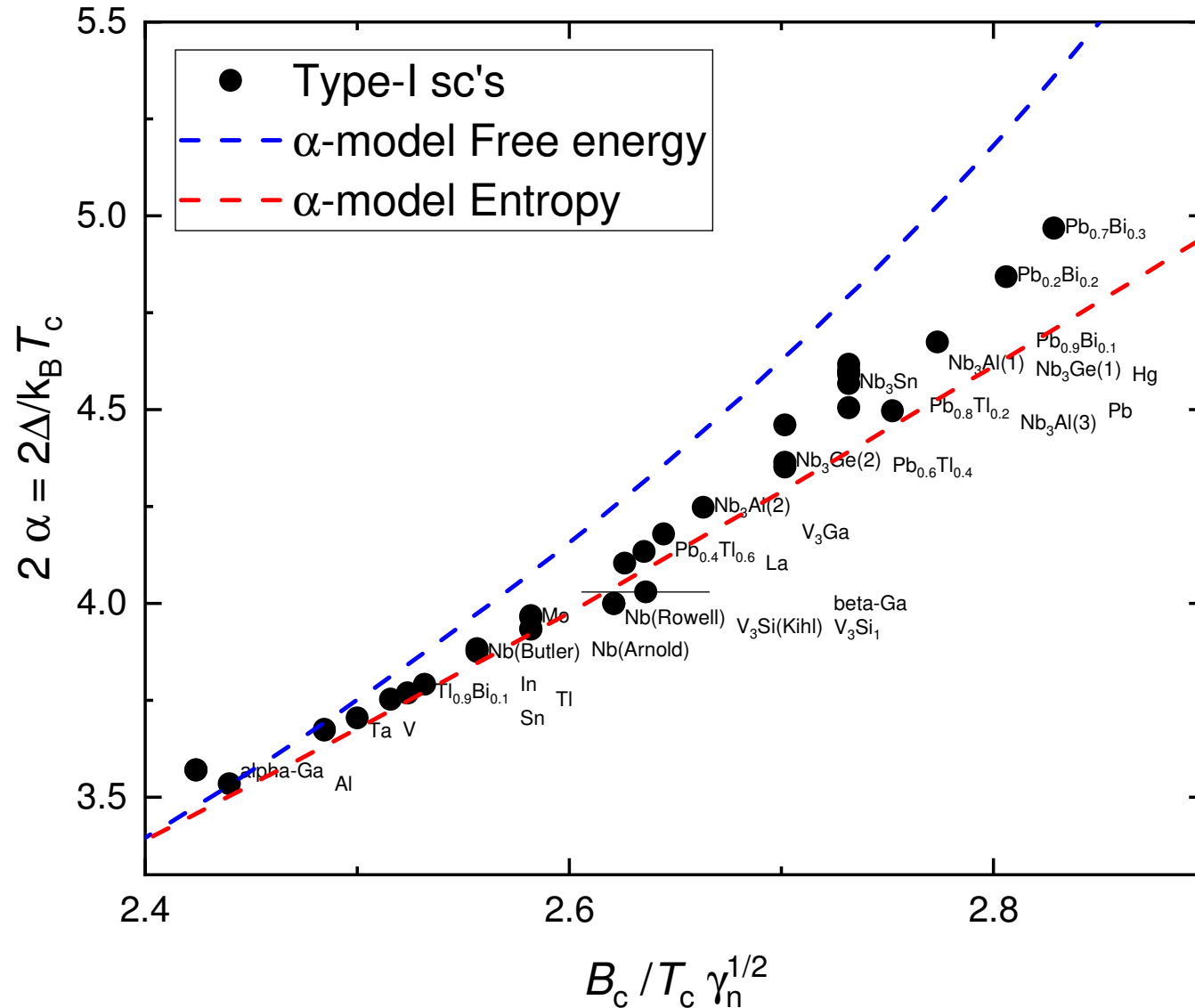
Universal relations for type-I superconductors

Universal relations for type-I superconductors



Kind of a “Uemura line” for conventional type-I superconductors!

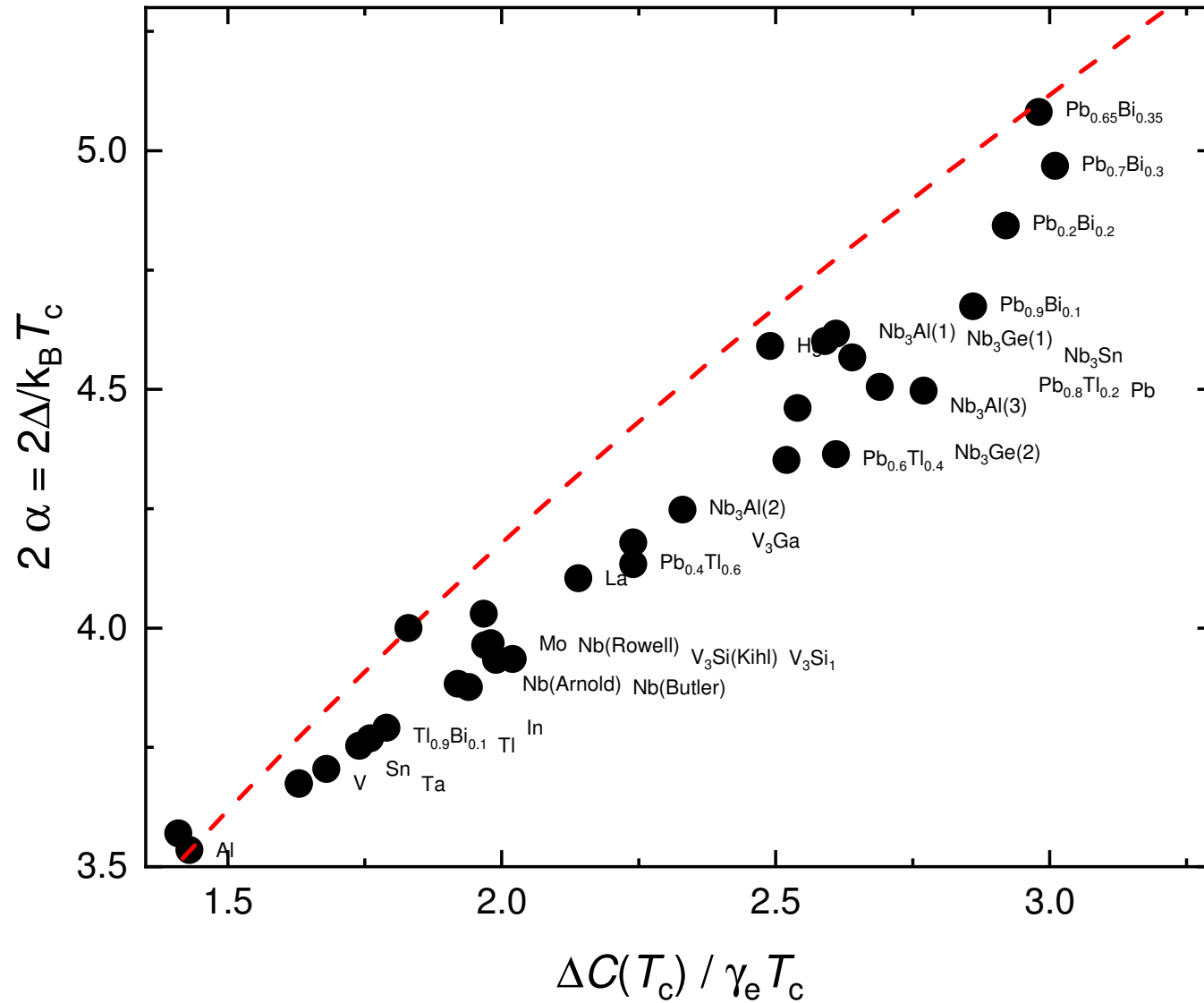
Universal relations for type-I superconductors




Species	Free electron value for γ in $mJmol^{-1}K^{-2}$	Experimental value for γ in $mJmol^{-1}K^{-2}$
Li	0.749	1.63
Be	0.500	0.17
Na	1.094	1.38
Mg	0.992	1.3
Al	0.912	1.35
K	1.668	2.08
Ca	1.511	2.9
Mn	170	5.2
Fe	4.98	10
Co	4.98	10.3
Cu	0.505	0.695
Zn	0.753	0.64
Ga	1.025	0.596
Rb	1.911	2.41
Sr	1.790	3.6
Ag	0.645	0.646
Cd	0.948	0.688
In	1.233	1.69
Sn	1.410	1.78
Cs	2.238	3.20
Ba	1.937	2.7
Au	0.642	0.729
Hg	0.952	1.79
Ti	1.29	1.47
Pb	1.509	2.98

In single-element superconductors γ is spread between 1 and 3!

Universal relations for type-I superconductors





Single element superconductors are still very interesting objects to be studied!

The universal trends for type-I superconductors become a consequence of a strong coupled BCS approach!

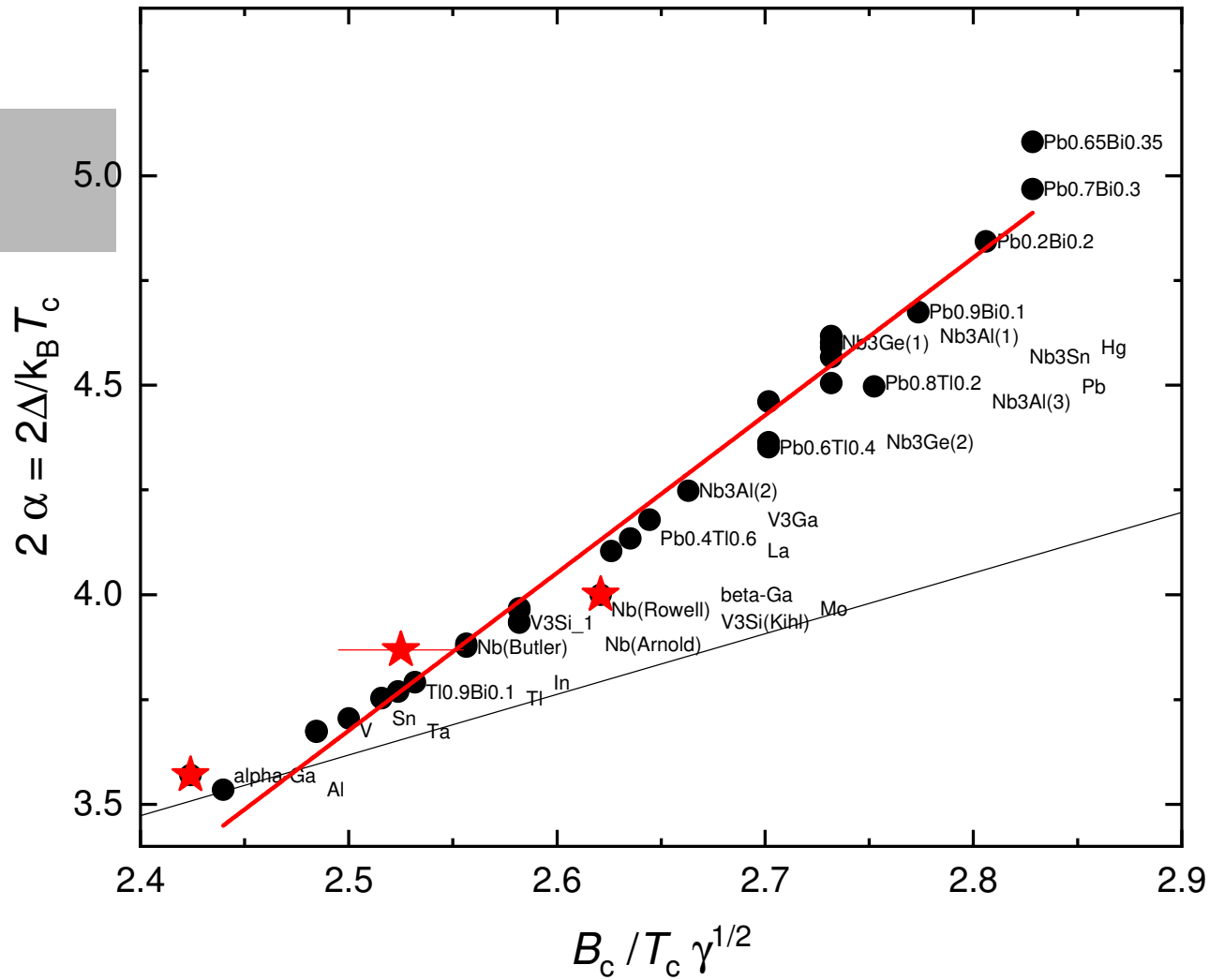


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α -model with free α

$$\frac{H_c(0)}{(\gamma_n T_c^2)^{1/2}} = \sqrt{\frac{6}{\pi}} \alpha \approx 1.382\alpha.$$

Black points from Carbotte RMP **62**, 1027 (1990)