Muons in superconductors

- Lesson I – the land we are exploring
  - Introduction: superconductivity, a story of three length-scales
  - London equations and the penetration depth
  - Ginzburg Landau equations and the coherence length
- Lesson II – the workhorse of μSR
  - The Abrikosov flux lattice
  - Muon determination of the penetration depth
  - Conventional and unconventional superconductivity: a glance
  - BCS: the gap and its temperature dependence
- Lesson III – material science
  - Clean vs. dirty superconductors
  - A phase diagram for superconducting materials
  - Towards atomic scale coherence: nanoscopic coexistence
  - Triplet superconductivity, topological superconductivity (?)

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  - Clean vs. dirty superconductors, extreme type II
  - A phase diagram for superconducting materials
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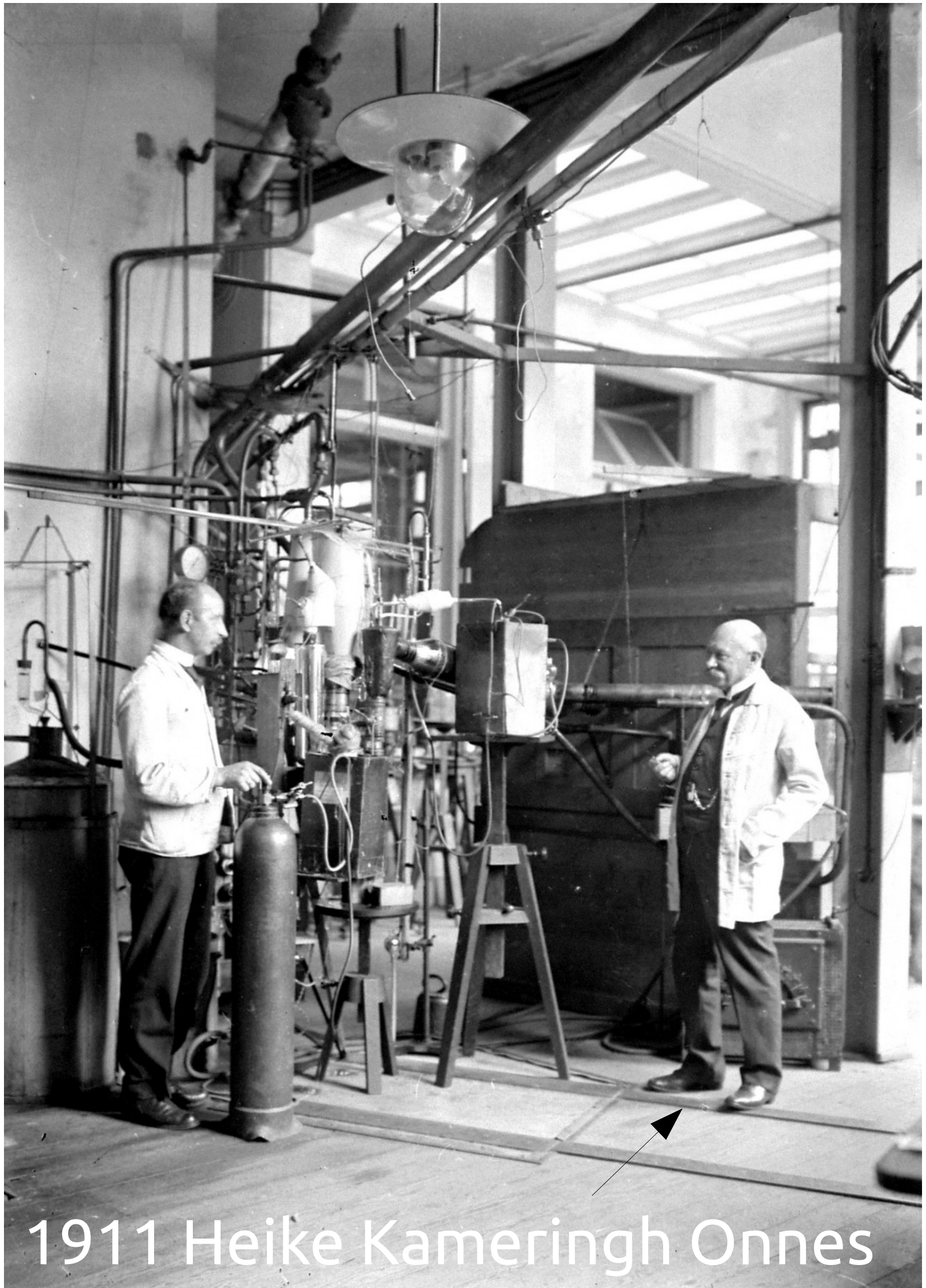
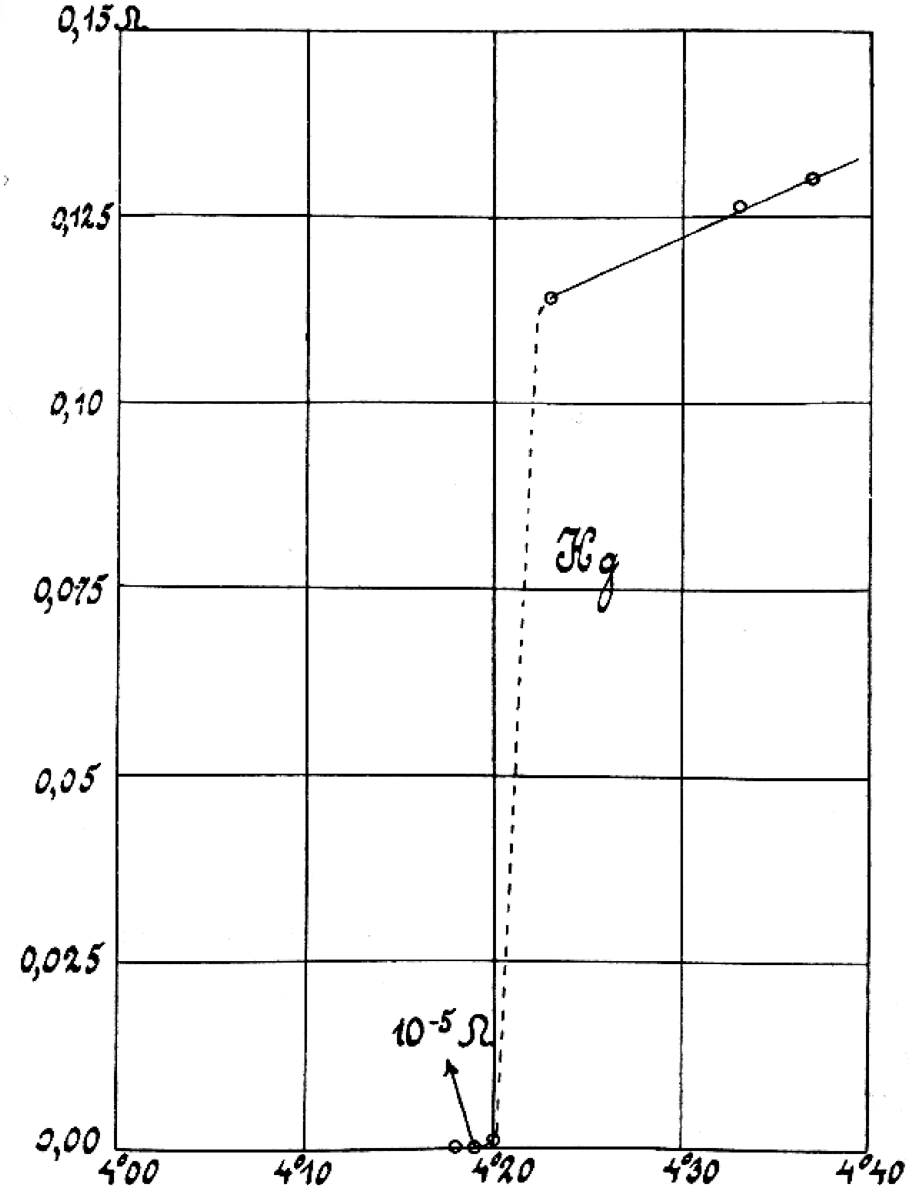
Introduction: Superconductivity





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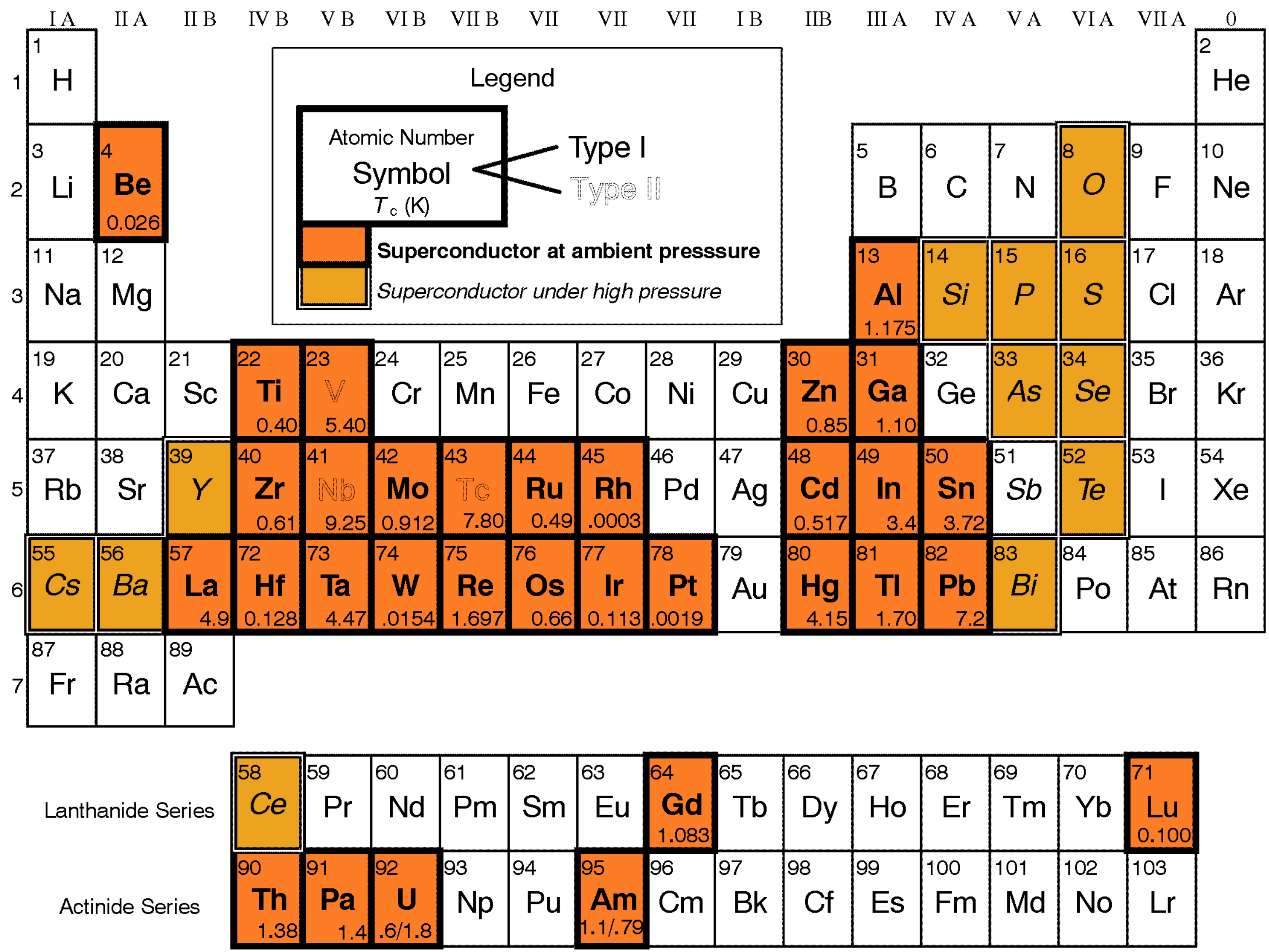
Elemental superconductors



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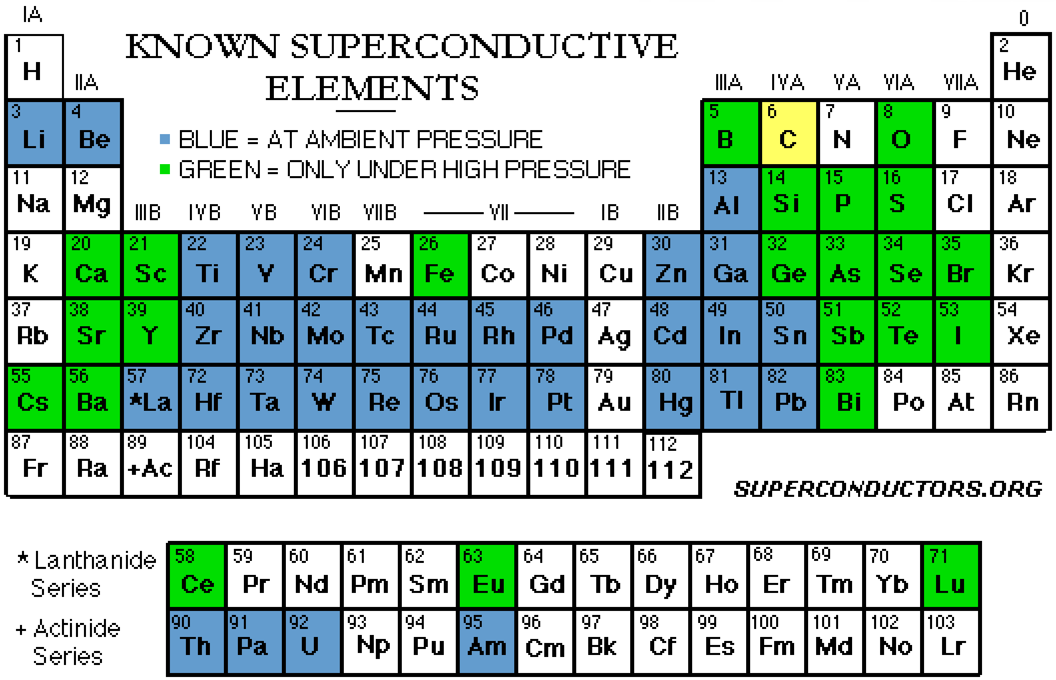
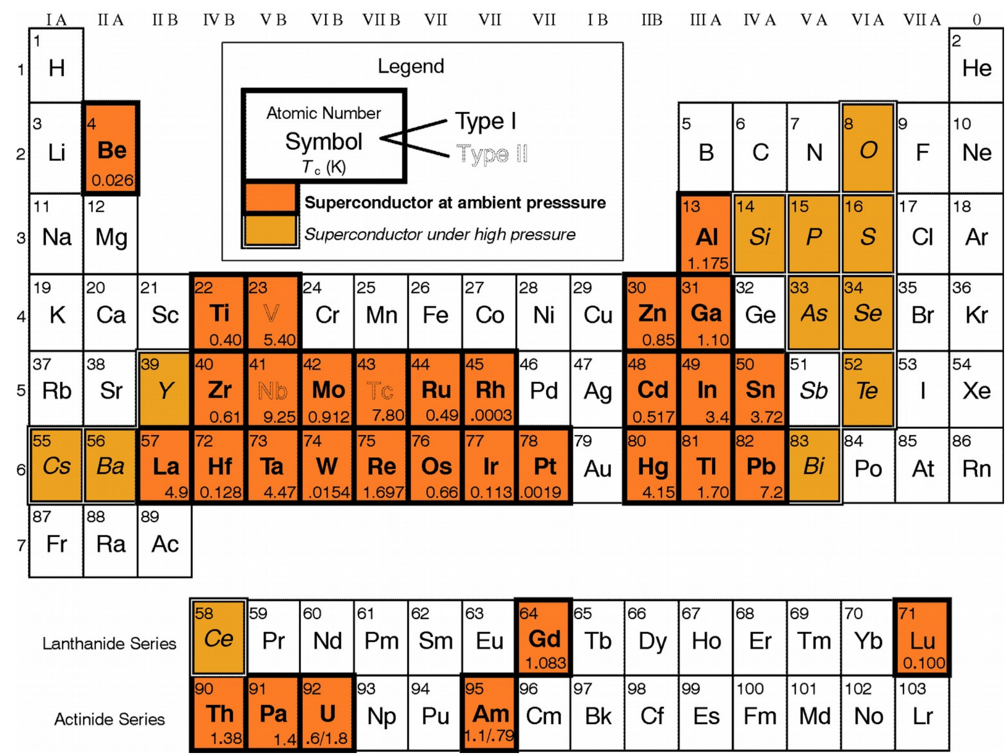
Elemental superconductors



http://www.superconductors.org/Type1.htm



Different sources: different shades of optimism

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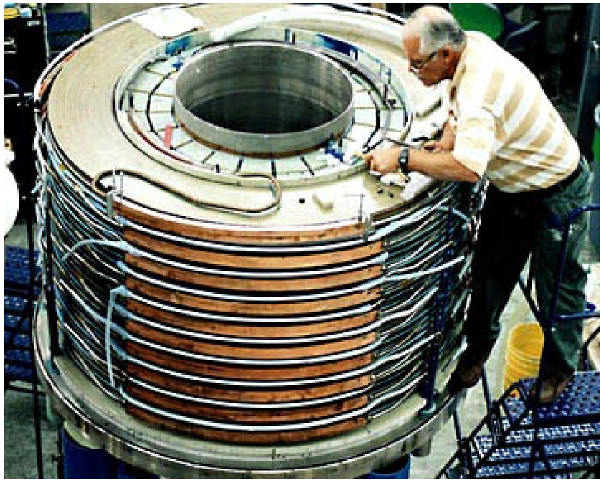
Why superconductors?

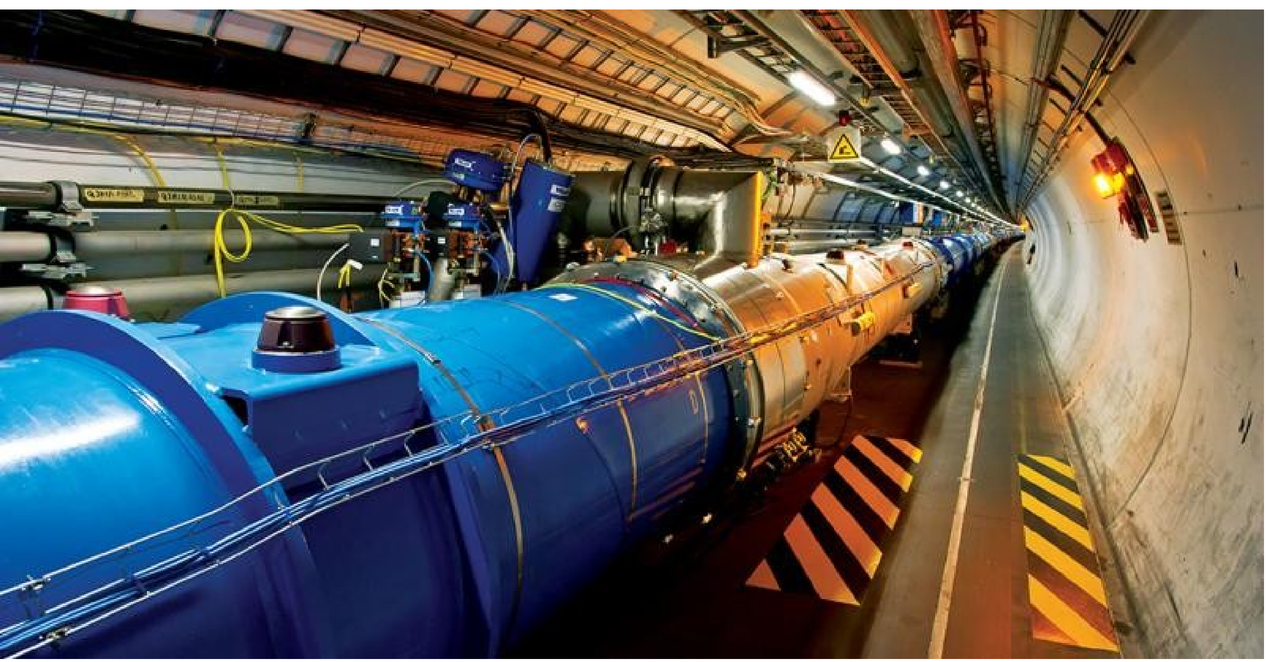


10 T conventional solenoid:

5 000 A in 1600 turns

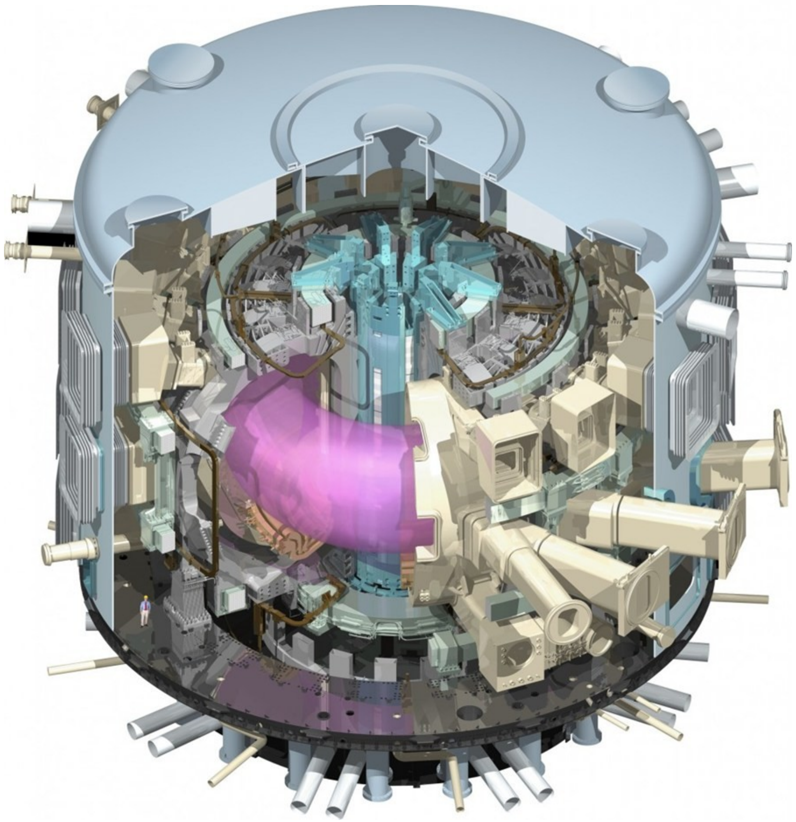
5 MW   homes in Abingdon

30 l liquid He/month



CERN
LHC
1232 main dipole
392 quadrupole
6000 corrector magnets

Why superconductors?

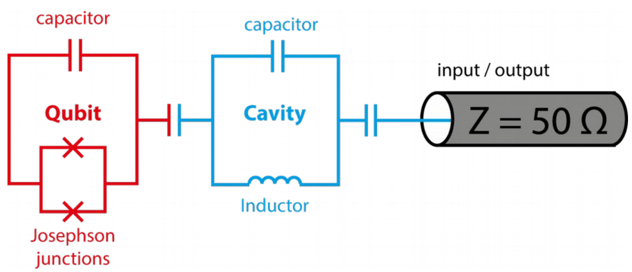


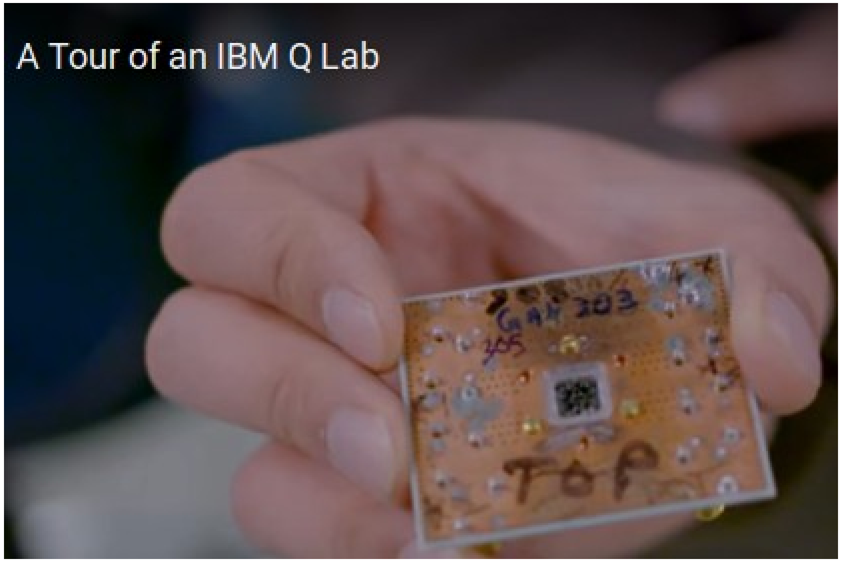
ITER tokamak

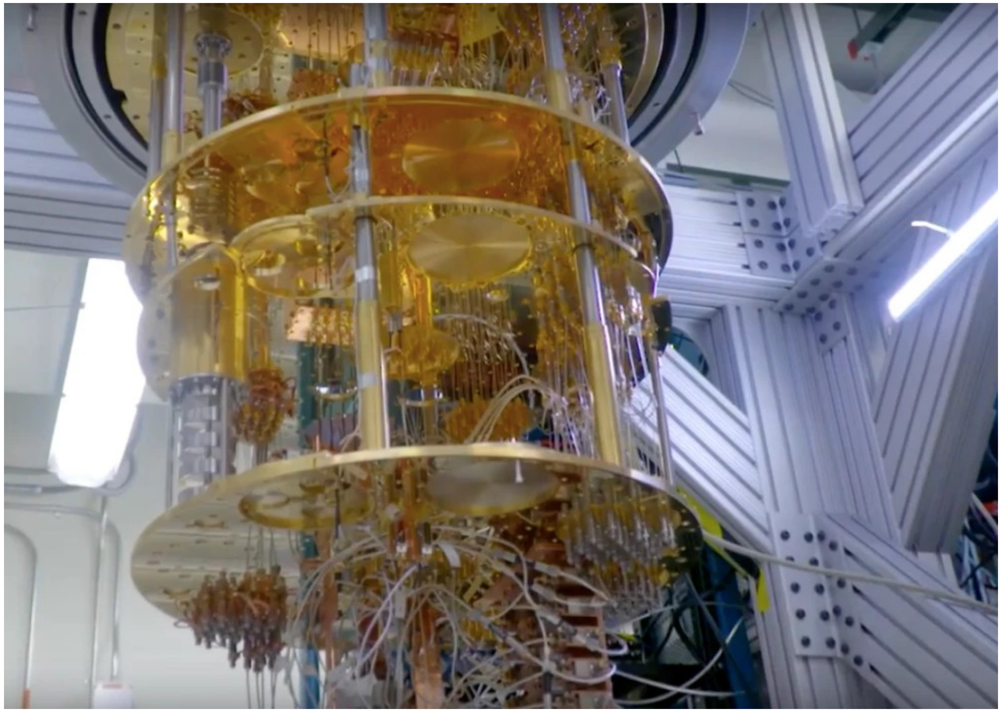
MRI

Why superconductors?

Quantum computation: transmons







$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Why superconductors?

A rare example of macroscopic quantum coherent state (with superfluids) $|\psi\rangle$

Also metals are an example of macroscopic quantum coherent state. $|\mathbf{k}\rangle$

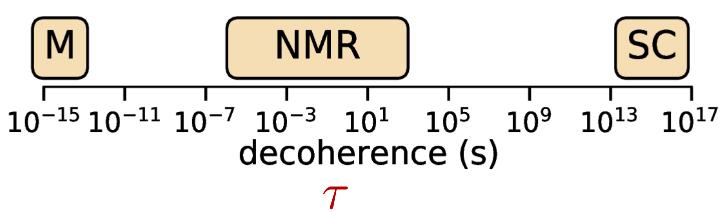
However

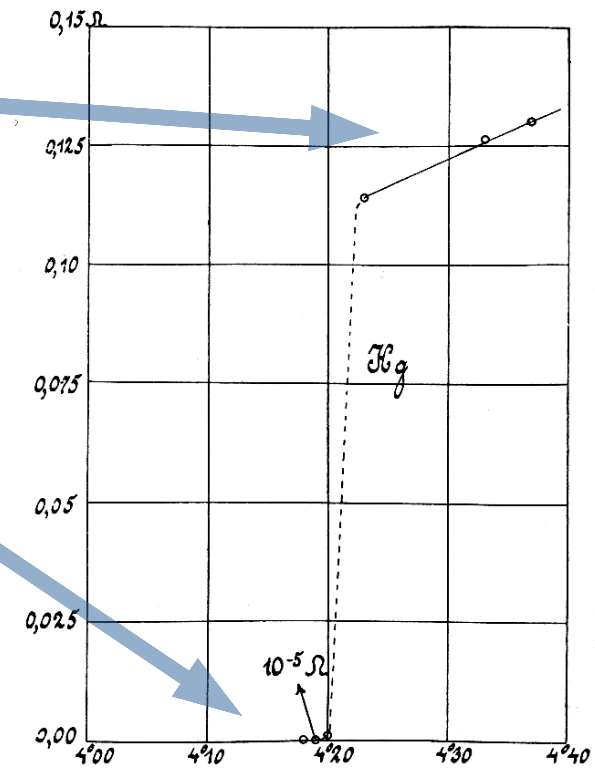
$$\rho = \frac{m}{ne^2\tau}$$

$$\tau \approx 10^{-15}\,\mathrm{s}$$

$$\rho = 0$$

$$\tau^* \approx 10^{17}\,\mathrm{s}$$





*decays in 10^{10} years
(not the same as Drude τ!)

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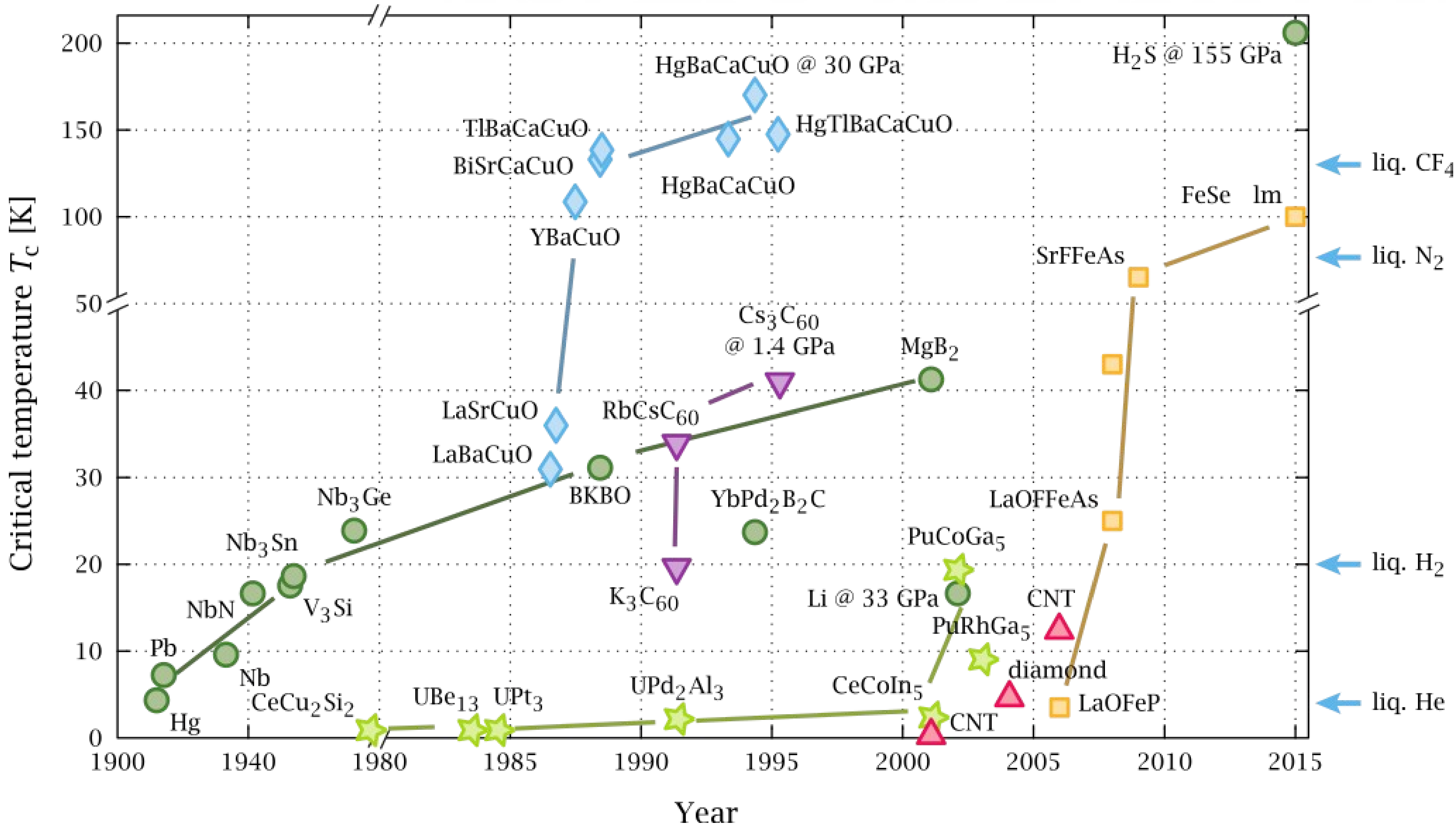
Why superconductors



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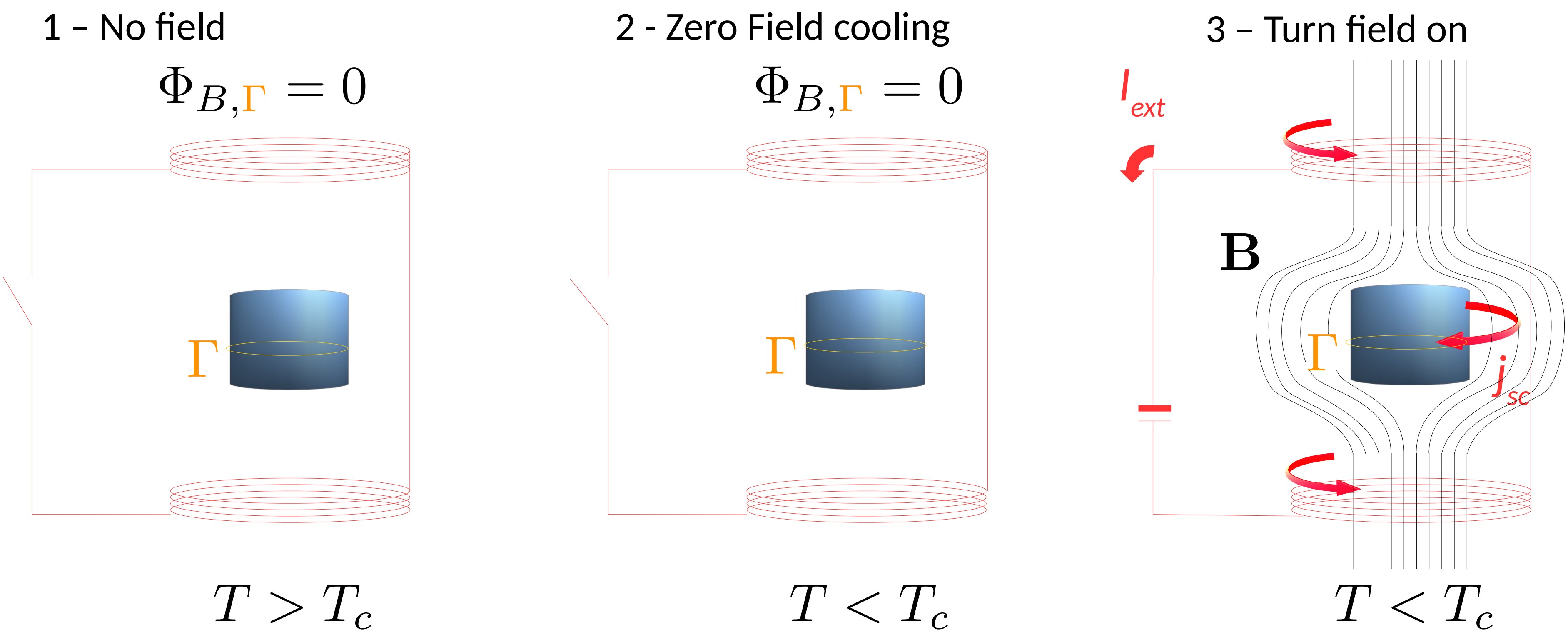
Persistent currents – 1
Perfect diamagnet

In a perfect conductor* $\rho = 0$

* with a phase transition at T_c

$$\frac{d\Phi_B}{dt} = 0$$

Shielding in three steps: 1 → 2 → 3



screening eddy-currents to keep $\Phi_{B,\Gamma} = 0$

This happen also in a superconductor $\rho = 0$

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Persistent currents – 2
Perfect diamagnet

In a perfect conductor $\rho = 0$

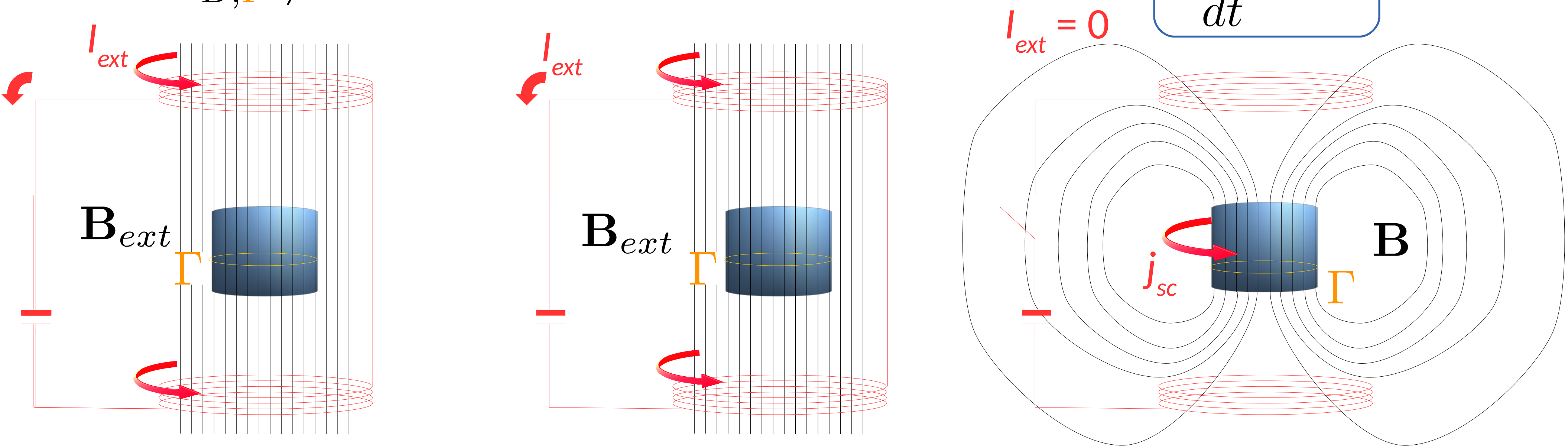
Establishing a persistent currents in three steps: 1 → 2 → 3

1 – Turn field on above T_c

2 – Field cooling

3 – Turn field off

$\Phi_{B,\Gamma} \neq 0$ $\Phi_{B,\Gamma} \neq 0$ $\frac{d\Phi_B}{dt} = 0$



$T > T_c$ $T < T_c$ $T < T_c$

This **does not** happen in a superconductor $\Phi_{B,\Gamma} \neq 0$

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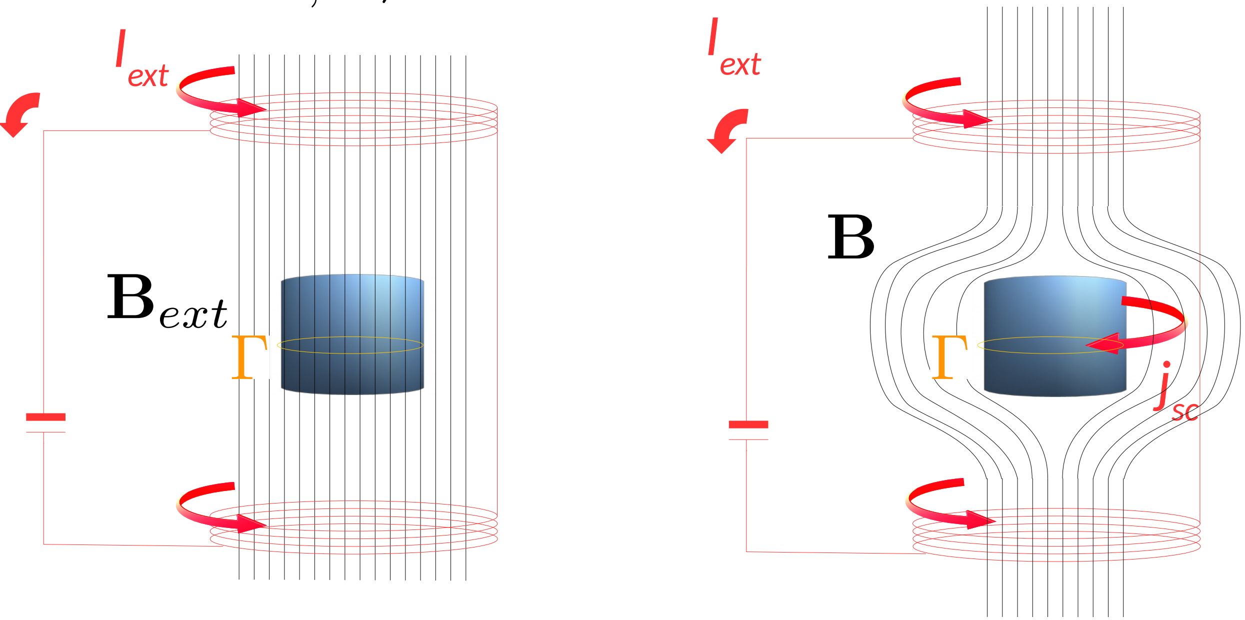
Persistent currents - 4

Meissner-Ochsenfeld effec: 1 → 2

1 – Set field above T_c 2 - Field cooling

$$\Phi_{B,\Gamma} \neq 0$$



$T > T_c$ $T < T_c$

Summary

A superconductor in an external field **B**, both F cooling and ZF cooling, expels the flux $\Phi_{B,\Gamma}$

The flux is expelled, so the real rule is

$$\Phi_{B,\Gamma} = 0$$

This would **not** happen to a perfect conductor

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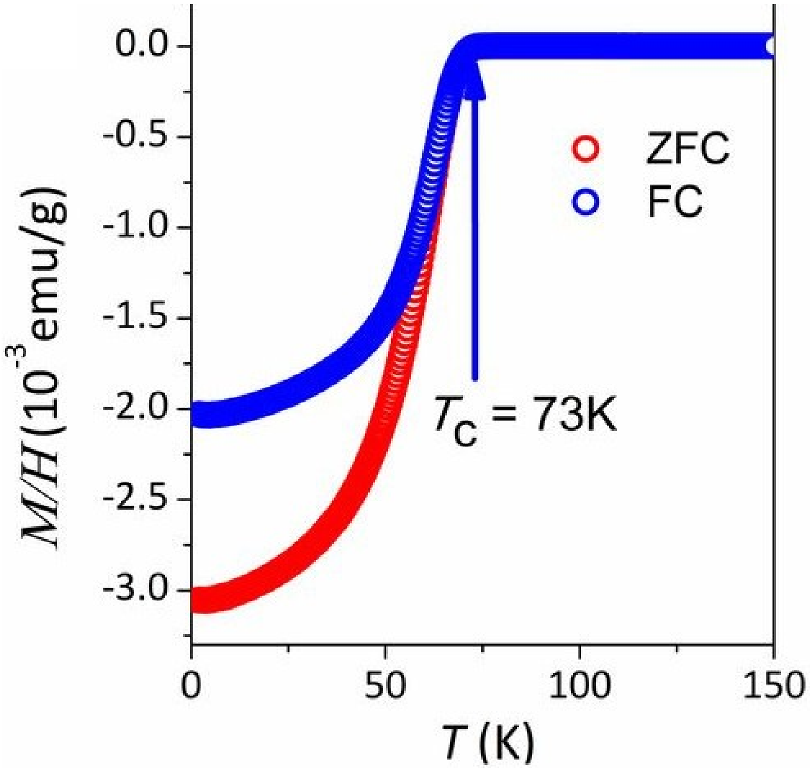
Field Cooling vs Zero Field Cooling

Negative M/H for ZFC is also
the response of a perfect conductor

Negative M/H in FC is the
signature of superconductivity

Extrinsic difference
due to flux pinning

Ba_2CuO_{4-y}



Zhao et al. PNAS 116 12156

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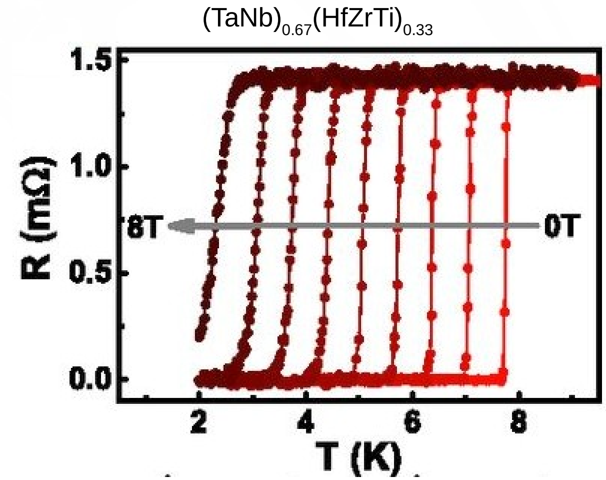
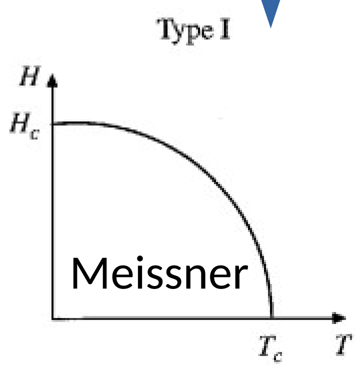
Type I and Type II superconductors

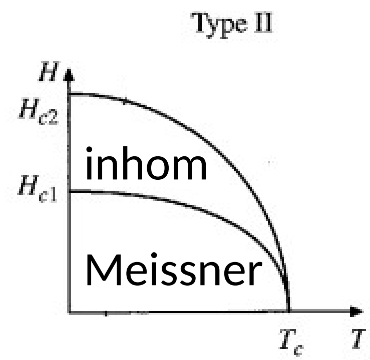


Jing Guo et al. PNAS 114, 13144

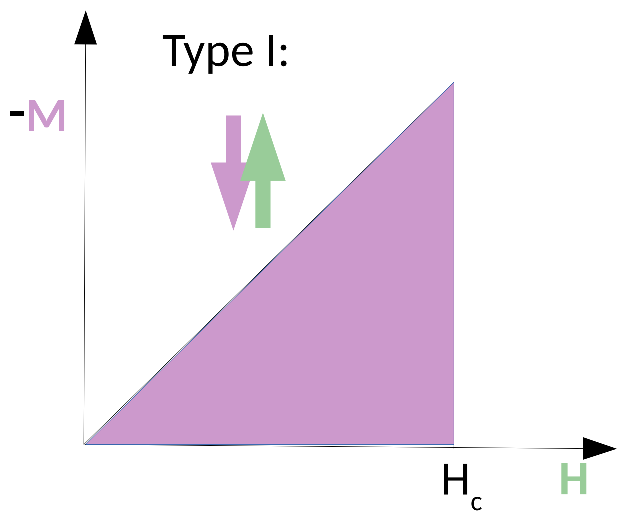
Critical field: superconductivity disappears for $H > H_c$

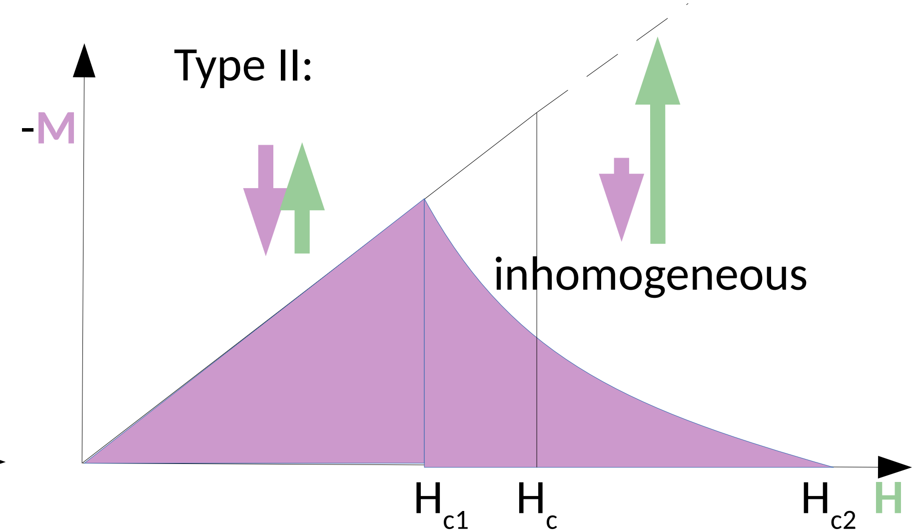
$$\chi = \left.\frac{dM}{dH}\right|_{H=0} = -1$$





$$|\chi| = \left.\frac{dM}{dH}\right|_{H=0} \ll 1$$





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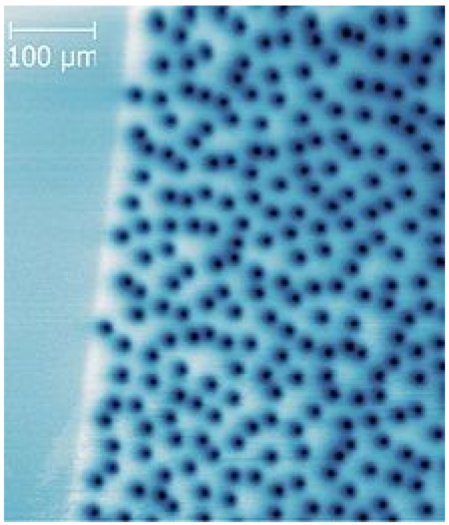
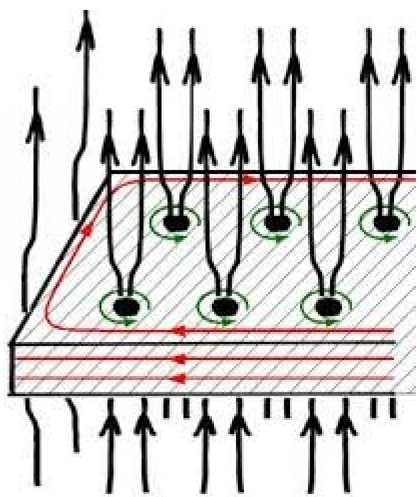
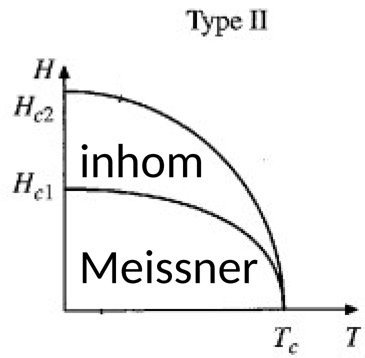
Type I and Type II

What inhomogeneity for $H > H_{c1}$?
(super)current vortices encircling quantized magnetic flux $\Phi_0 = \frac{h}{2e}$

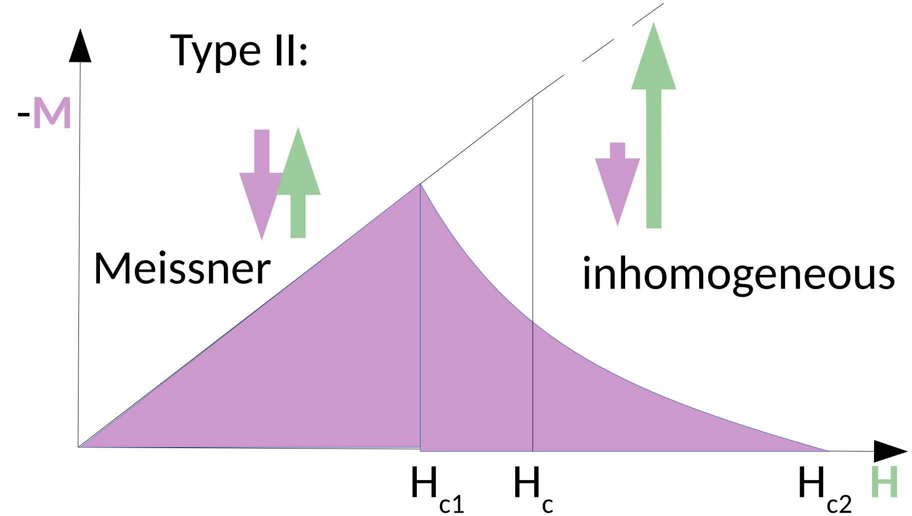


vortices in YBCO
imaged by scanning
SQUID microscopy





$$|\chi| = \frac{dM}{dH}\bigg|_{H=0} \ll 1$$



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Three length-scales

- London penetration depth
  - λ controls the magnetic field penetration
- Coherence length
  - ξ controls the quantum coherence of the ground state
- Mean free path
  - ℓ controls scattering

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London equation

Sketch of deep argument on electron wavefunction:

- incoherent in normal Drude metal $\mathbf{J} = nq\mathbf{v}$

Fritz London, 1900-1956

No power supply

$$\langle \mathbf{p} \rangle = 0 \rightarrow m\langle \mathbf{v} \rangle = 0 \rightarrow \langle \mathbf{J} \rangle = 0$$

- quantum coherent in superconductors

Superconducting state $\langle \mathbf{p} \rangle = 0$ even after switching fields on.

Minimal substitution $m\mathbf{v} = \mathbf{p} - e\mathbf{A}$

$$\langle \mathbf{v} \rangle = -\frac{e}{m}\mathbf{A} \rightarrow \mathbf{J}_s = -\frac{ne^2}{m}\mathbf{A}$$

London equation

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London penetration depth

$$\mathbf{J}_s = -\frac{ne^2}{m}\mathbf{A}$$ London equation

Substituting in Ampére law one obtains

$$\nabla^2\mathbf{B} = \frac{\mu_0 ne^2}{m}\mathbf{B}$$

$$\frac{1}{\lambda^2}$$

For London

after Cooper pairs

electron mass → m ← $2m$

$$\lambda = \left(\frac{m}{\mu_0 ne^2}\right)^{\frac{1}{2}}$$

London penetration depth

electron density → n ← $\frac{n}{2}$

electron charge → e^2 ← $(2e)^2$

Also

$$\nabla^2\mathbf{A} = \frac{\mu_0 ne^2}{m}\mathbf{A}$$

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London penetration depth derivation

$$\mathbf{J}_s = -\frac{ne^2}{m}\mathbf{A}$$ London equation

take the curl of the stationary Ampére law $\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}$

Vector identity

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{B} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \left(-\frac{ne^2}{m} \boldsymbol{\nabla} \times \mathbf{A} \right)$$

By Gauss law $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$

$$\lambda = \left(\frac{m}{\mu_0 ne^2} \right)^{\frac{1}{2}}$$

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}$$

Magnetic field (London approximation)

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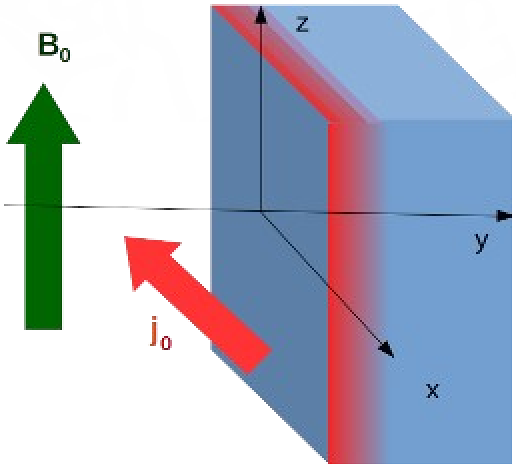
What does λ imply?

Ampere law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$



semi-infinite slab

Guess the solution

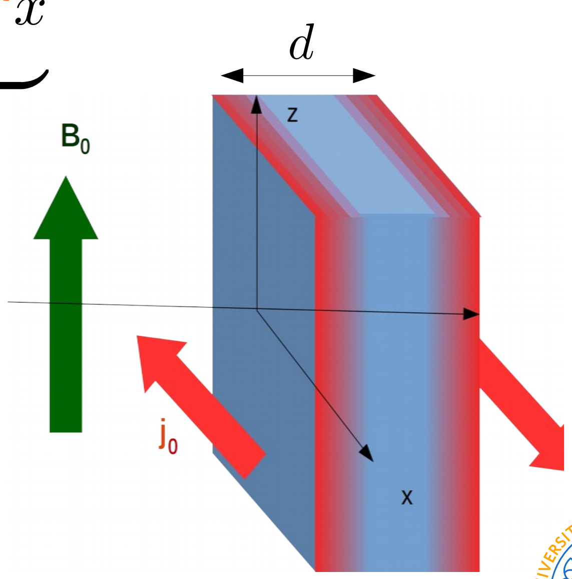
$$\mathbf{B} = B_0 e^{-y/\lambda} \hat{z}$$

Right!

$$\nabla \times \mathbf{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & B_0 e^{-y/\lambda} \end{vmatrix} = \underbrace{-\frac{B_0}{\lambda} e^{-y/\lambda} \hat{x}}_{\mu_0 \mathbf{j}}$$

Thin sample

$$\mathbf{B}(x,y,z) = \mu_0 k_0 \frac{\cosh(y/\lambda)}{\cosh(d/\lambda)} \hat{z}$$



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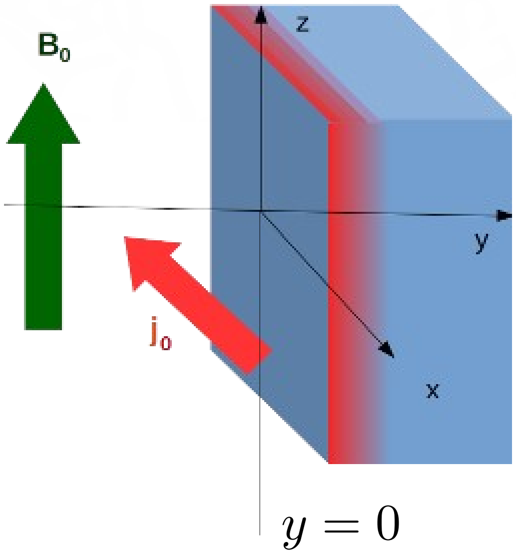
Exercise: do it properly



Check that

$$\mathbf{B} = \mu_0 k_0 \hat{z} \begin{cases} 1 & y < 0 \\ e^{-y/\lambda} & y > 0 \end{cases}$$

and

$$\mathbf{A} = -\mu_0 k_0 \hat{x} \begin{cases} y & y < 0 \\ \lambda e^{-y/\lambda} & y > 0 \end{cases}$$

are solutions of

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}$$

and

$$\nabla^2 \mathbf{A} = \frac{1}{\lambda^2} \mathbf{A}$$

with $\mathbf{J}_s = -\frac{ne^2}{m} \mathbf{A}$

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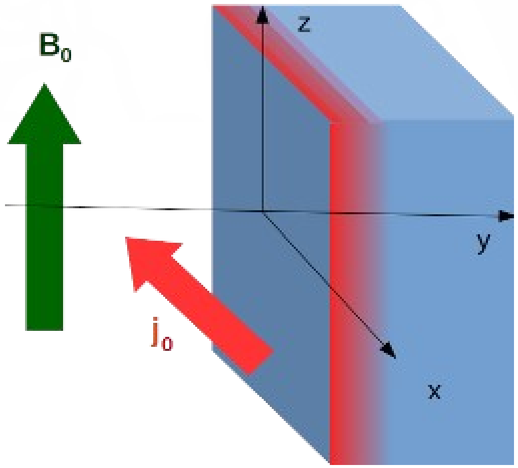
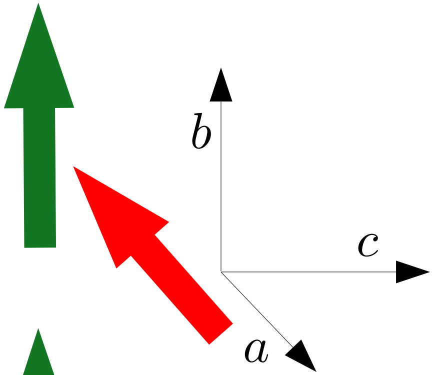
Anisotropic metals

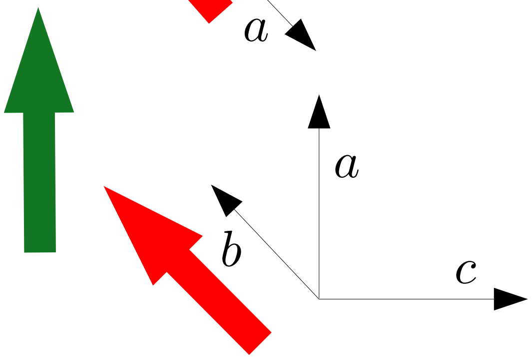
$$\lambda^2 = \frac{m}{\mu_0 n e^2} \rightarrow \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_c \end{bmatrix} \frac{1}{\mu_0 n e^2}$$



$$\lambda_a^2 = \frac{m_a}{\mu_0 n e^2}$$



$$\lambda_b^2 = \frac{m_b}{\mu_0 n e^2}$$



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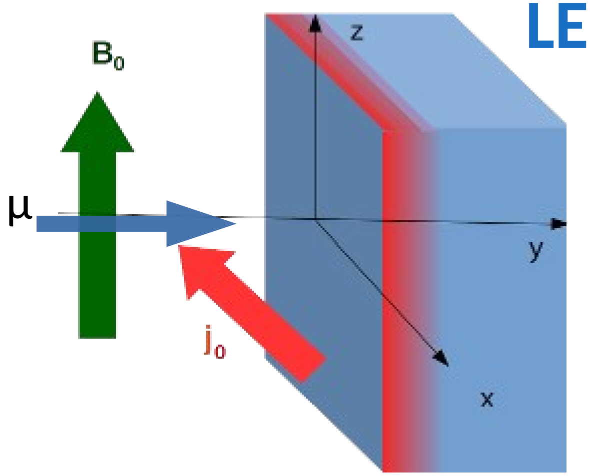
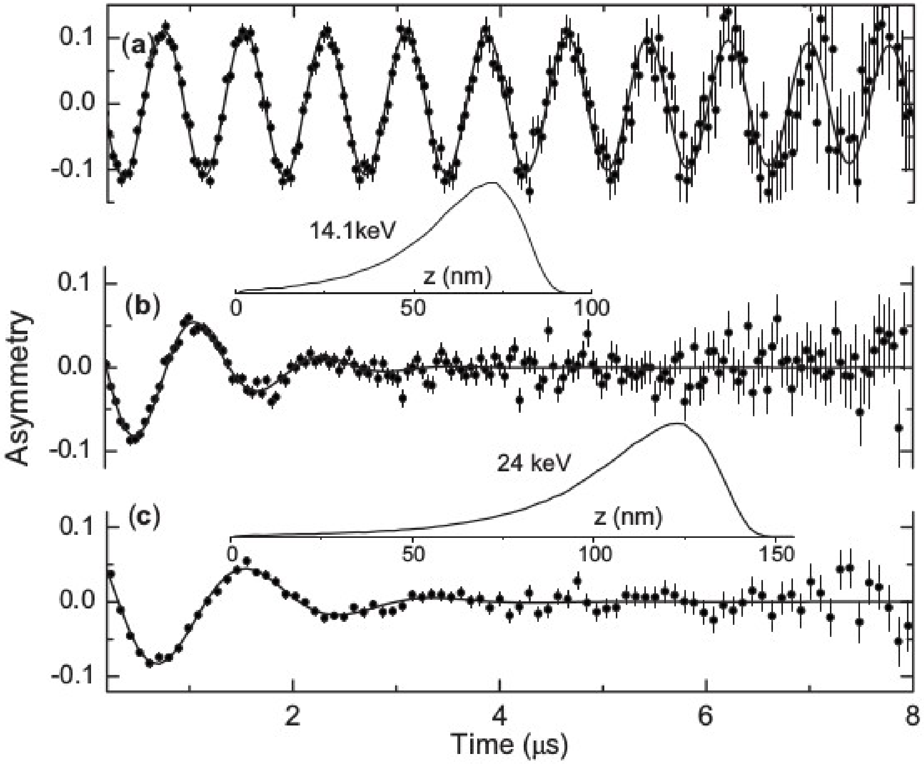
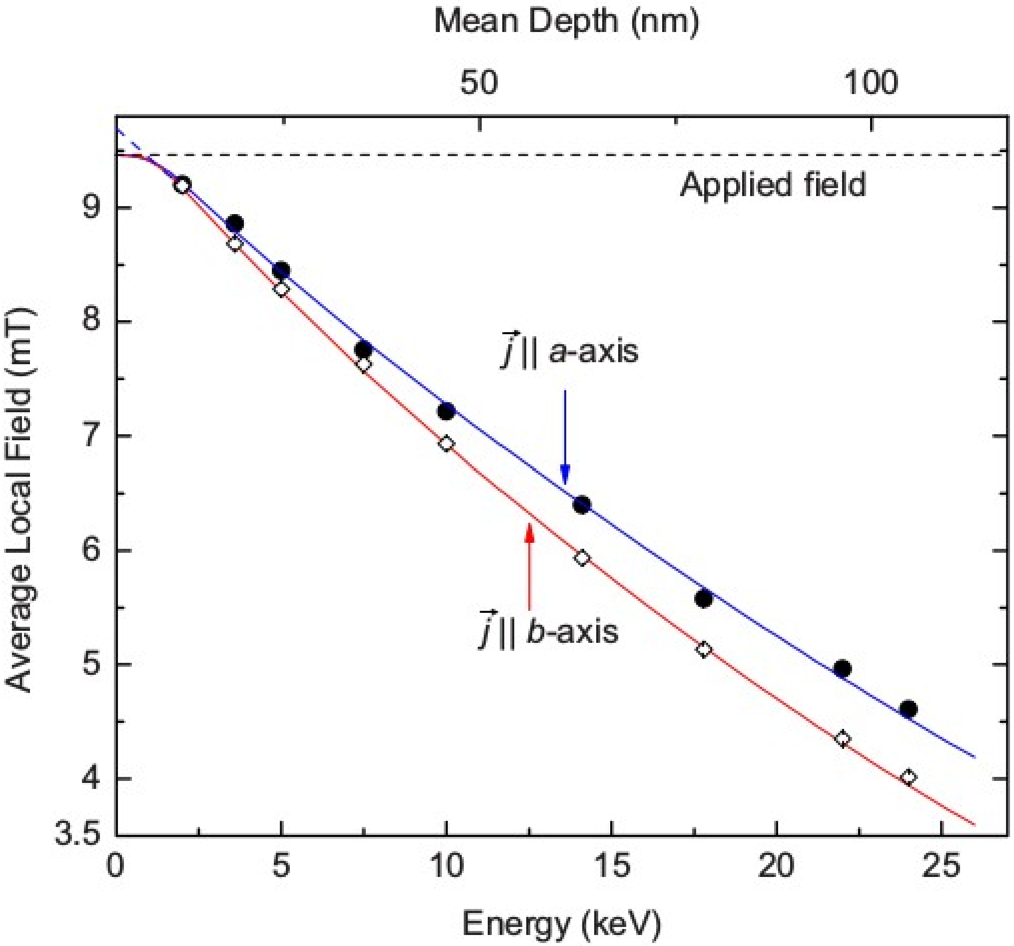
LEM experiment



Kiefl et al . Phys. Rev. B. 81 180502





Landau model

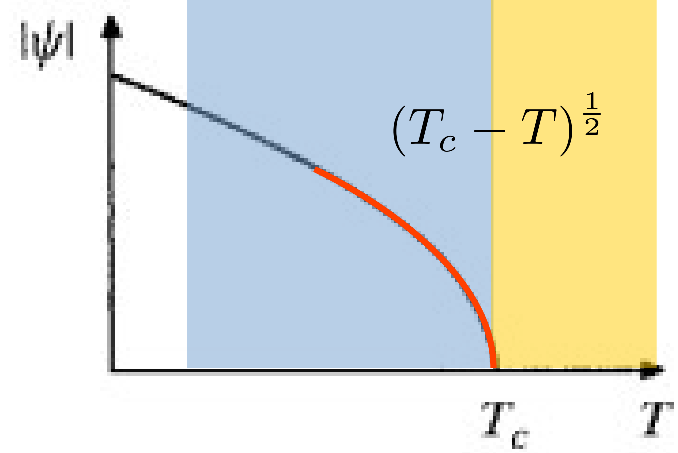
The order parameter is a complex function ψ and the free energy density is

$$f_s(H) = f_n(H) + a(T - T_c)|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\mu_0}{2}HM$$

linearise!

For $a, b > 0$ (only below T_c and below B_c)

$$|\psi|^2 = \frac{a}{b}(T_c - T) \qquad = 0$$



Condensation energy ≡ maximum energy that supercurrents can expell,
corresponds to a tiny free energy density

$$f_n(H_c) - f_s(H_c) = \frac{\mu_0}{2}H_c^2 = \frac{1}{2\mu_0}B_c^2$$

Compare

$$v_{cell}\frac{B_c^2}{2\mu_0} \approx 1\,\mu\text{eV}$$

$$\epsilon_F \approx 1\,\text{eV}$$

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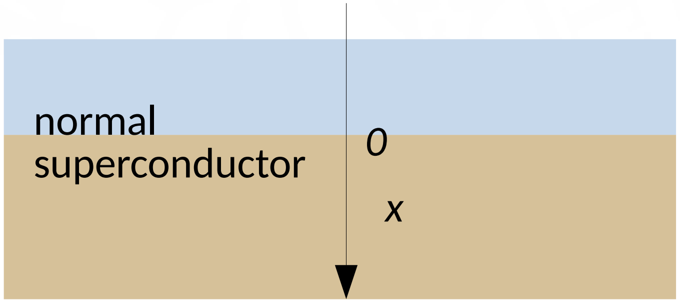
Ginzburg-Landau coherence length

Inhomogeneous order parameter



In zero *B* field, linearised (*b* = 0)

$$f_s(0) = f_n(0) + a(T - T_c)|\psi|^2 + \boxed{\frac{\hbar^2}{2m}|\nabla\psi|^2}$$

Ginzburg-Landau free energy density

cost of varying the order parameter

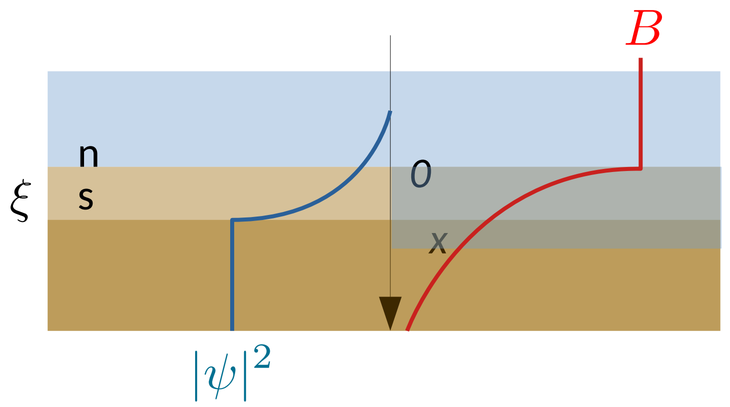
The ratio of the order parameter to the gradient term is a square lengthscale

$$\left[\frac{|\psi|^2}{|\nabla\psi|^2}\right] = \boxed{\frac{\hbar^2}{2ma|T - T_c|} = \xi^2}$$

energy loss

$\propto B_c^2 \xi$

condensation energy



energy gain

$\propto B_c^2 \lambda$

(shielding)

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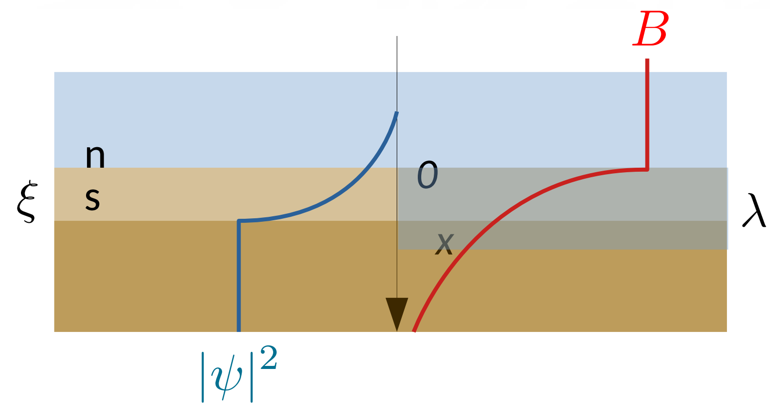
Type-I vs Type-II again

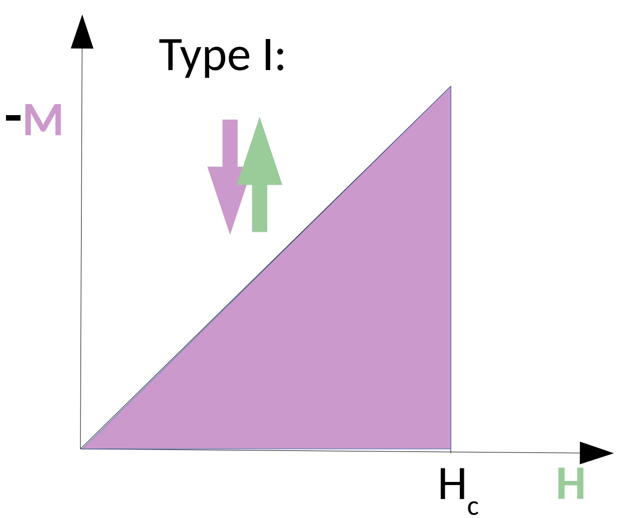
energy loss
$\propto B_c^2 \xi$
condensation energy



energy gain
$\propto B_c^2 \lambda$
(shielding)

Homogeneous superconductor interfaces cost energy

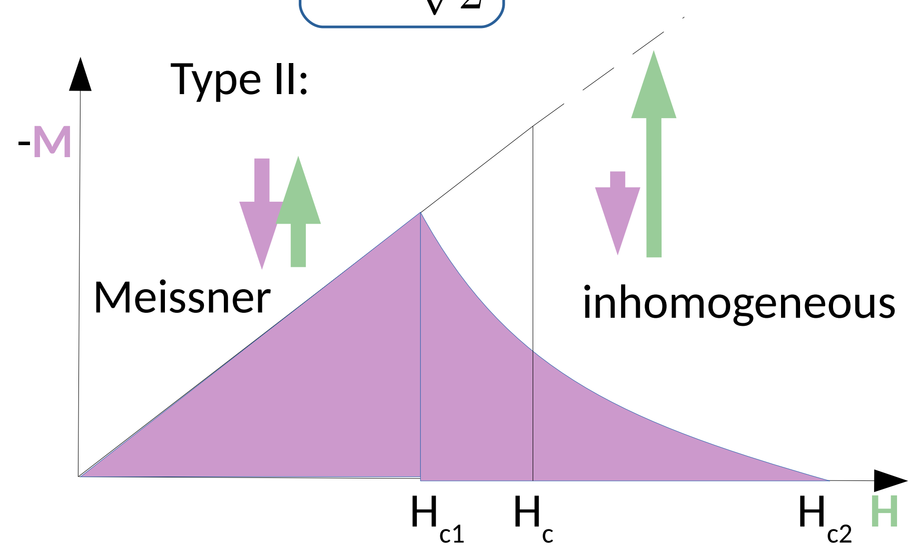
$$\kappa < \frac{1}{\sqrt{2}}$$



$$\kappa = \frac{\lambda}{\xi}$$

It is convenient to have normal-superconductor interfaces

$$\kappa > \frac{1}{\sqrt{2}}$$



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GL equations

Minimizing the free energy with respect to $\nabla\psi$

$$f = a(T-T_c)|\psi|^2 + \frac{1}{2m^*}\left|\frac{\hbar}{i}\nabla\psi\right|^2$$

GL (linearised) equation

$$-\frac{\hbar^2\nabla^2}{2m^*}\psi = a(T_c - T)\psi$$

: like a Schrödinger equation

In a magnetic field

Minimizing the free energy with respect to $\nabla\psi, \mathbf{A}$ independently

$$f = a(T-T_c)|\psi|^2 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla + 2e\mathbf{A}\right)\psi\right|^2 + \frac{1}{2\mu_0}|\nabla\times\mathbf{A}|^2$$

Two GL (linearised) equation

$$\left[-\frac{\hbar^2}{2m^*}\left(\nabla + i\frac{e^*}{\hbar}\mathbf{A}\right)^2 + a(T-T_c)\right]\psi = 0$$

$$\mathbf{J} = -\frac{e^*\hbar}{i2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{(e^*)^2}{m^*}|\psi|^2\mathbf{A}$$

$$\mathbf{J}_s = -\frac{ne^2}{m}\mathbf{A}$$

cfr. London

Single London vortex

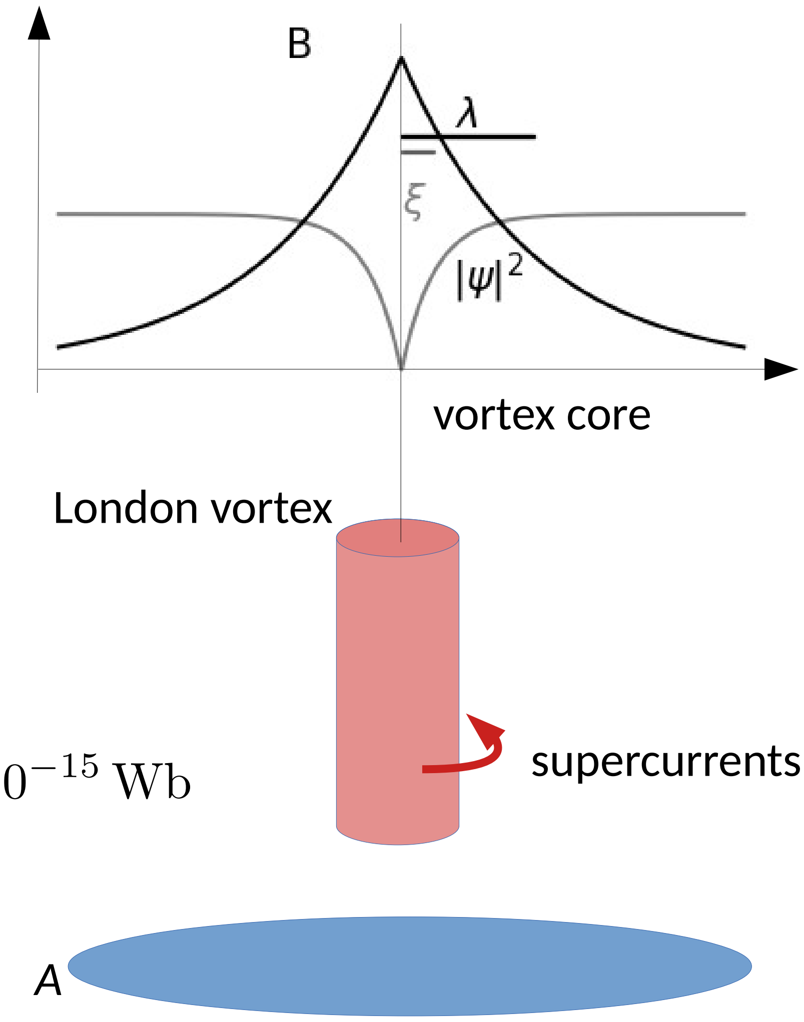
It is convenient to have normal-superconductor interfaces when $$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$$

May be convenient to have *field defects*

$$\nabla^2 \mathbf{B} - \frac{1}{\lambda^2}\mathbf{B} = \Phi_0 \delta(\mathbf{r})$$

Field must be quantized!

1 fluxon $$\int_A B \cdot d\mathbf{a} = \Phi_0 = \frac{h}{2e} = 2.0678 \cdot 10^{-15}\,\mathrm{Wb}$$



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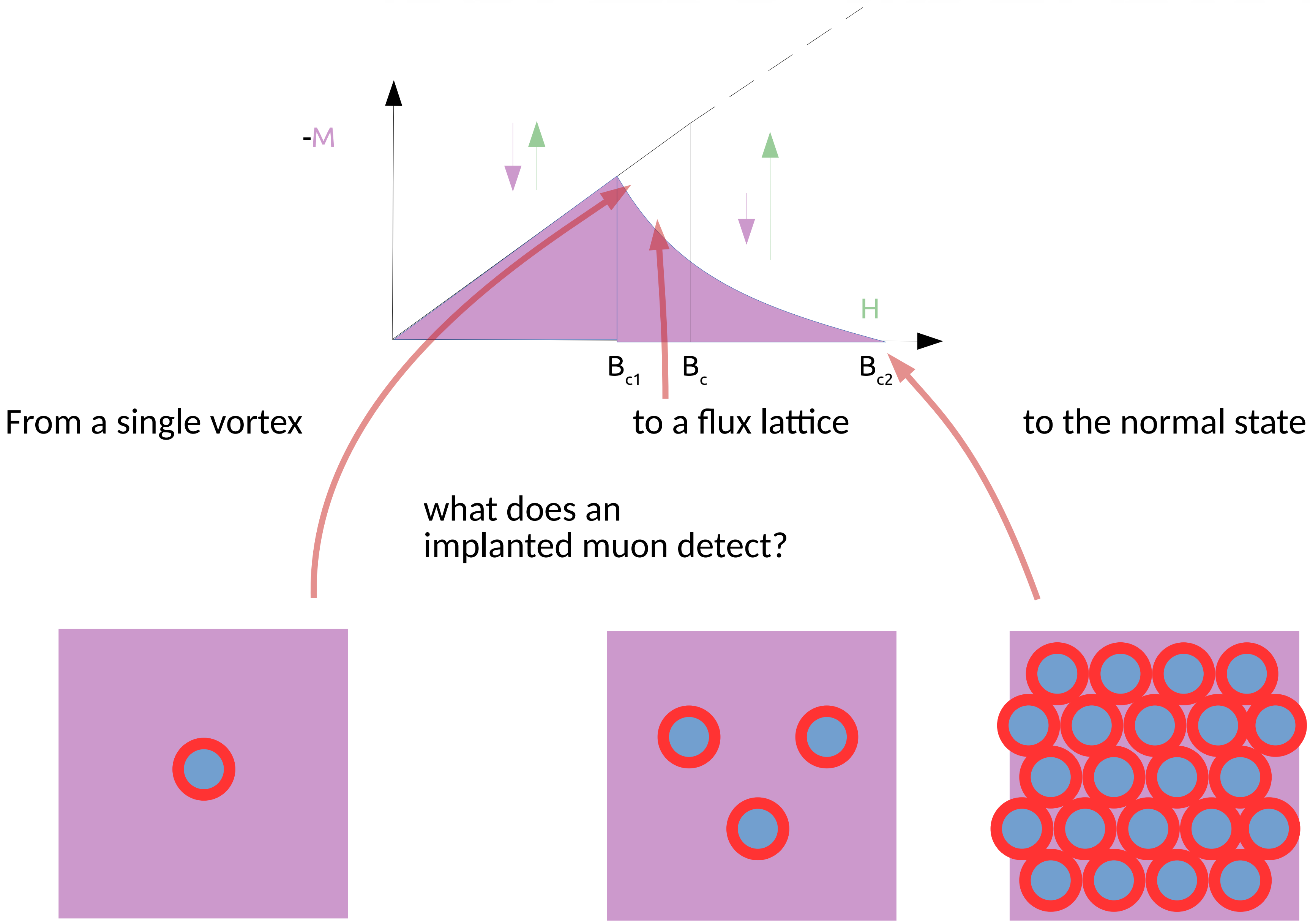
Next lecture: type II



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