Muons in superconductors

- Lesson I – the land we are exploring
  - Introduction: superconductivity, a story of three length-scales
  - London equations and the penetration depth
  - Ginzburg Landau equations and the coherence length
- Lesson II – the workhorse of μSR
  - The Abrikosov flux lattice
  - Muon determination of the penetration depth
  - Conventional and unconventional superconductivity: a glance
  - BCS: the gap and its temperature dependence
- Lesson III – material science
  - Clean vs. dirty superconductors, extreme type II
  - A phase diagram for superconducting materials
  - Towards atomic scale coherence: nanoscopic coexistence
  - Triplet superconductivity, topological superconductivity (?)

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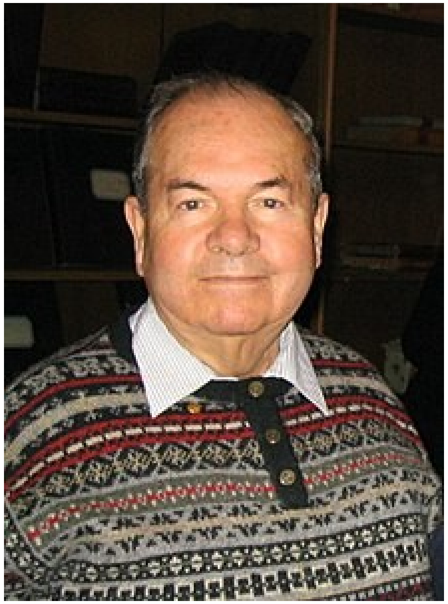
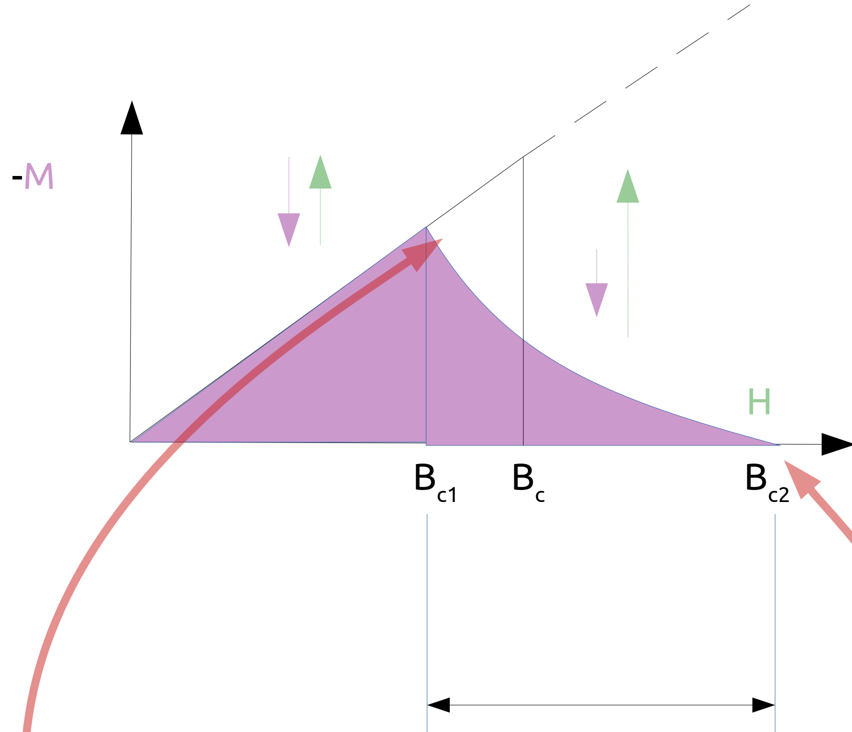
Type II superconductors



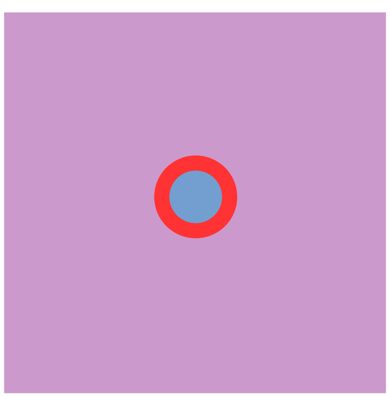
Alexei A. Abrikosov
Nobel Prize 2003



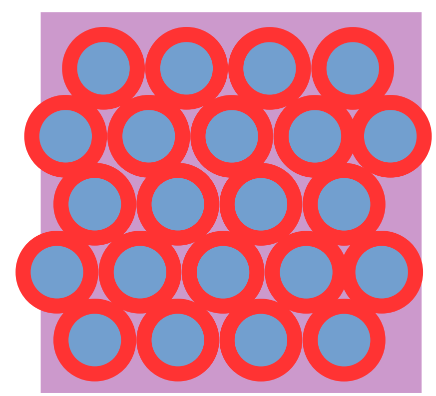
Area = condensation energy density

$$B_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$

Vortex (flux line) lattice



found as an ingegnous solution of linearised GL equations



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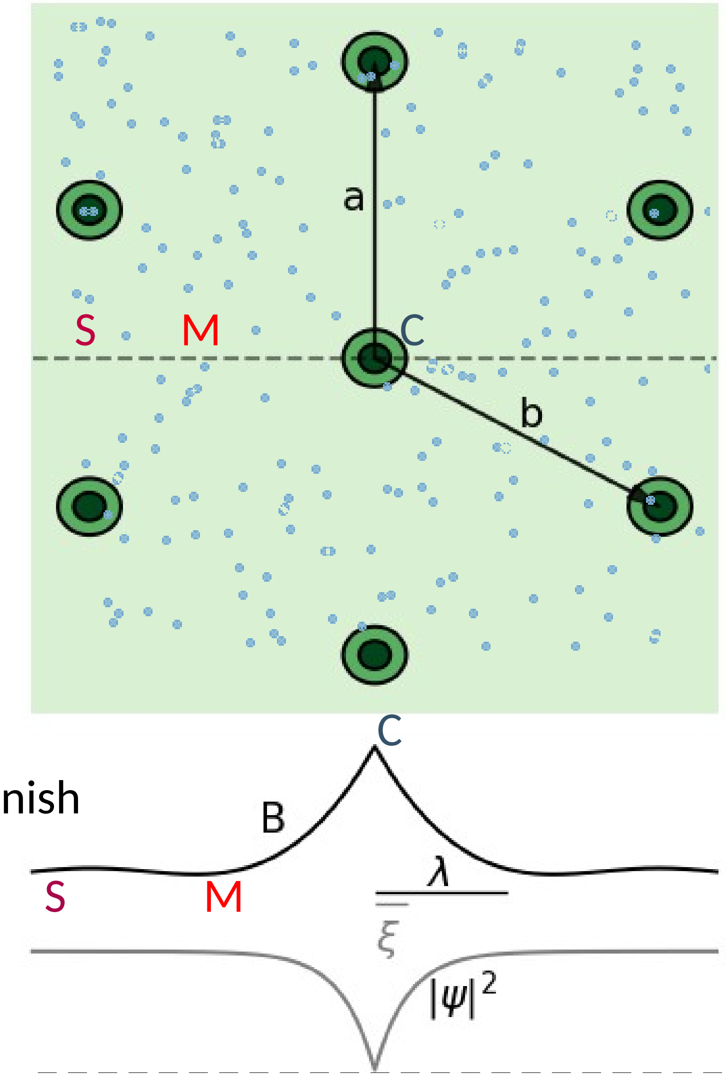
The flux line lattice

Most often triangular (or square) free energy minimum

$$a_{\triangle} = b_{\triangle} = \left(\frac{4}{3}\right)^{\frac{1}{4}} \left(\frac{\Phi_0}{B}\right)^{\frac{1}{2}}$$

incommensurate to crystal lattice (much larger)



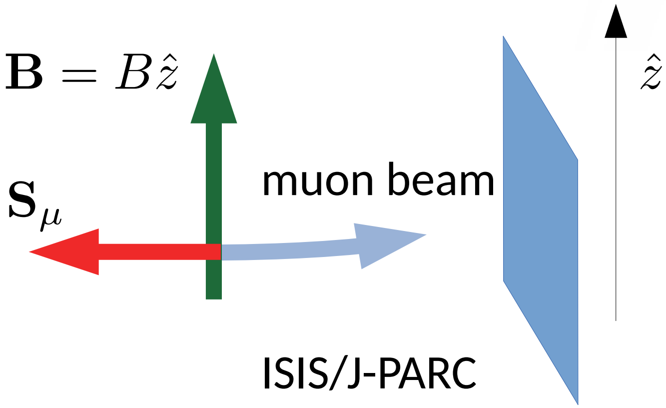
Field does not vanish
due to overlap

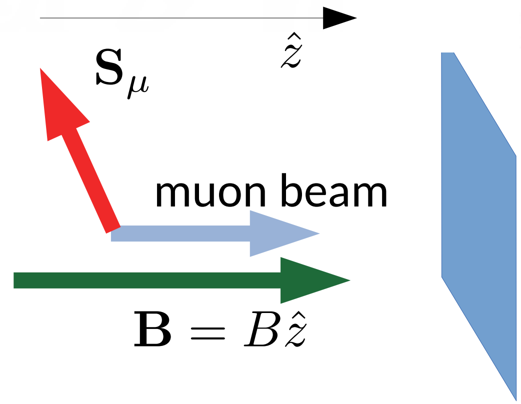
order parameter vanishes
at the core centre

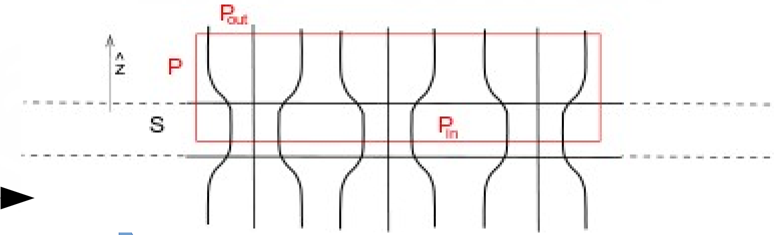
Muons sample uniformly the field distribution $p(B)$
TF-µSR asymmetry:

$$A(t) = A_0 \int p(B) \cos(\gamma_\mu B t + \phi) dB$$

Measure TF asymmetry A(t)



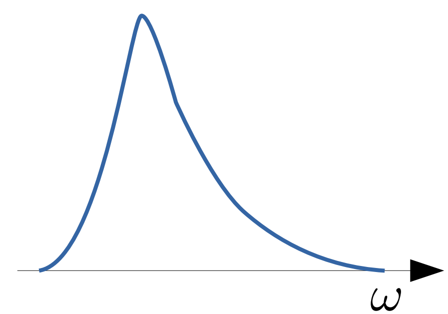




away from sample surface $\mathbf{B}(\mathbf{r}) = B(\mathbf{r})\hat{z}$

$$A(t) = A_0 \int p(B)\cos(\gamma_\mu B t + \phi)dB$$

How do you get $p(B)$ from the asymmetry?



Fourier transform

$$p(B) = p\left(-\frac{\omega}{\gamma_\mu}\right) = \frac{1}{A_0}\int A(t)\cos(\omega t + \phi)dt$$

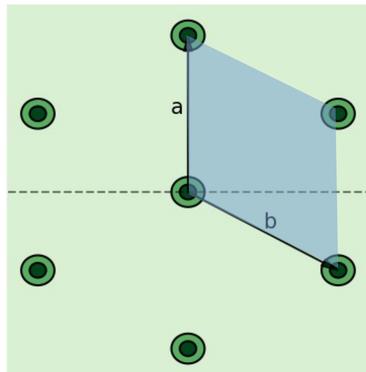
phase tuning!

Field distribution of a vortex lattice

London model

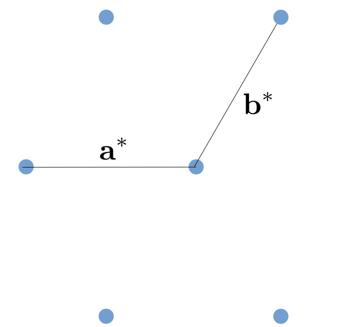
$$\nabla^2 \mathbf{B} - \frac{1}{\lambda^2}\mathbf{B} = \Phi_0 \hat{z} \sum_{\mathbf{R}} \delta(\mathbf{R})$$

$$\mathbf{R} = n\mathbf{a} + m\mathbf{b}$$



$$\langle B \rangle = \frac{\Phi_0}{|\mathbf{a} \times \mathbf{b}|}$$

$$\mathbf{Q} = n\mathbf{a}^* + m\mathbf{b}^*$$



$$|\mathbf{a}^*| = \frac{2\pi}{|\mathbf{a}|}$$

Solution by (Fast) Fourier Transform

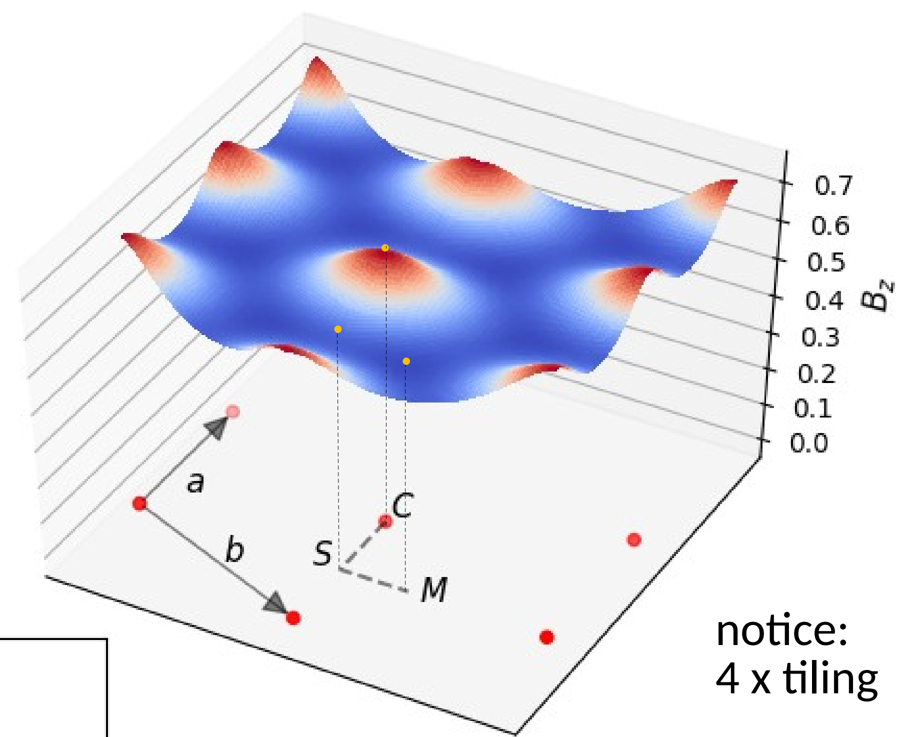
$$B(\mathbf{r}) = \langle B \rangle \sum_{\mathbf{Q}} b(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}}$$

$$b(\mathbf{Q}) = \frac{1}{1 + \lambda^2 Q^2}$$

The field distribution by μSR

$$B(\mathbf{r}) = \langle B \rangle \sum_{\mathbf{Q}} b(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}}$$

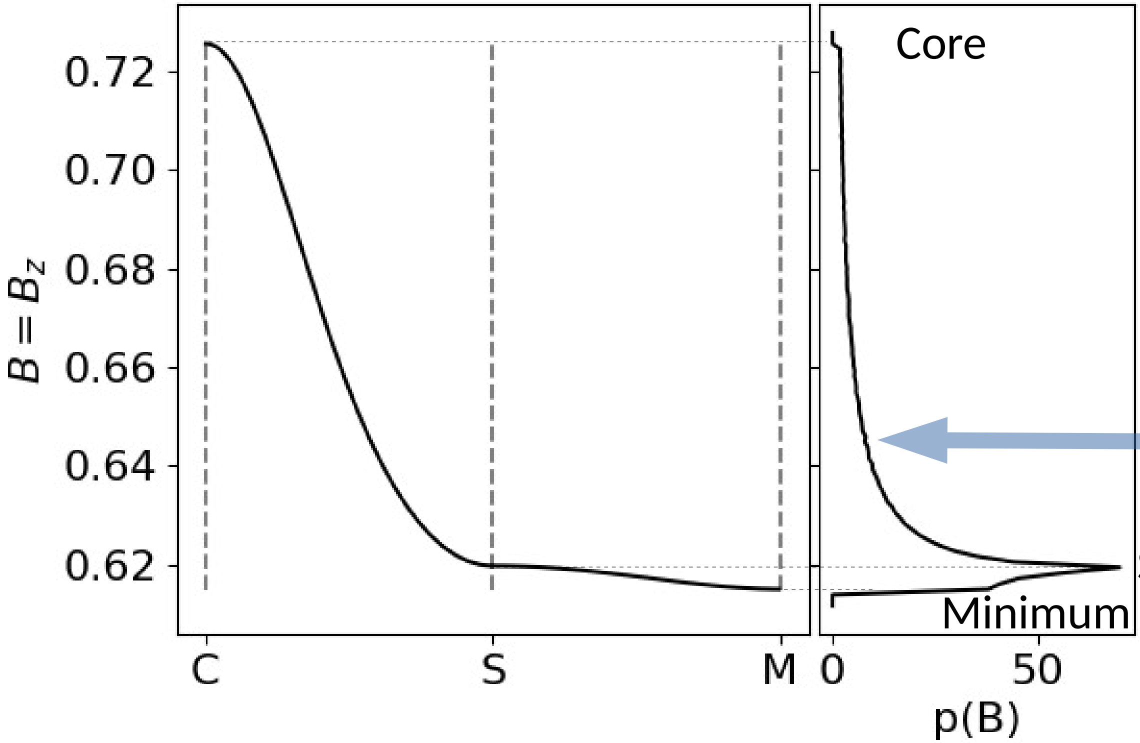
$$b(\mathbf{Q}) = \frac{1}{1+\lambda^2 Q^2}$$



notice:
4 x tiling

$$p(B) = \int dS_{\mathbf{r}} \left| \frac{dB}{d\mathbf{r}} \right|^{-1}$$

```
Bq = B(Q)
Br = fft(Bq)
pB = histogram(Br)
```



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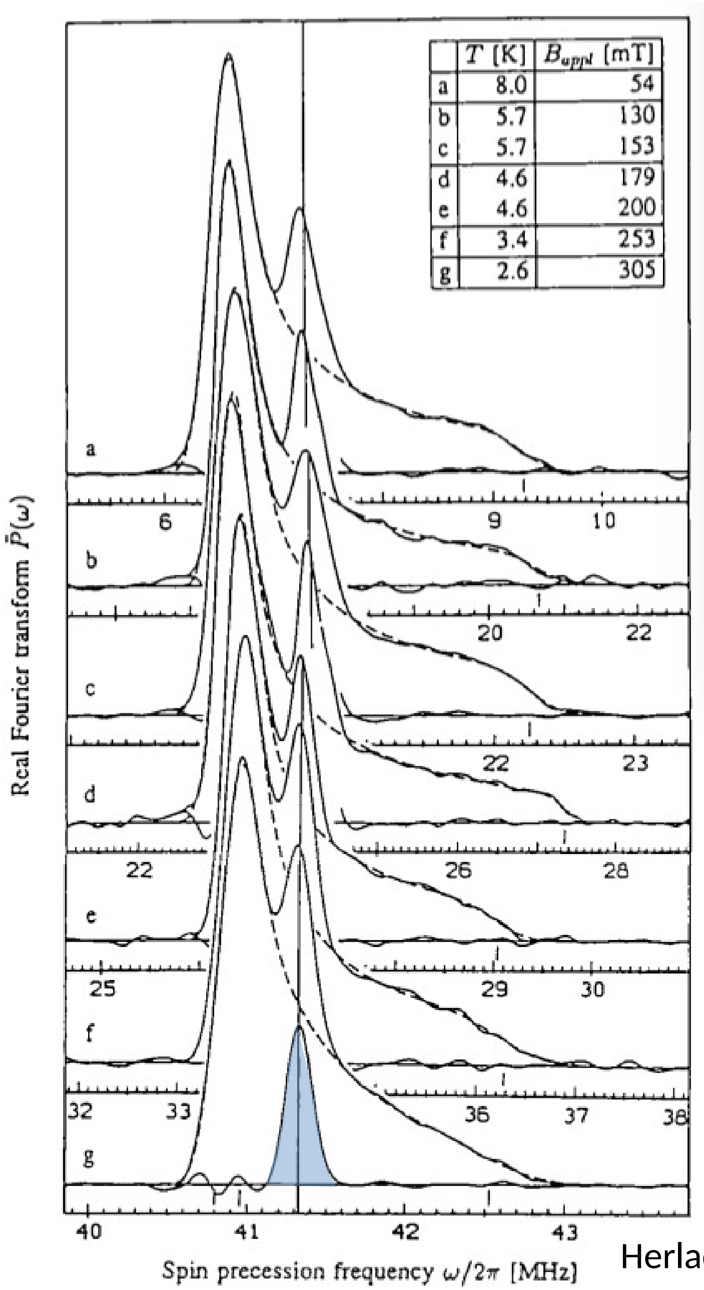
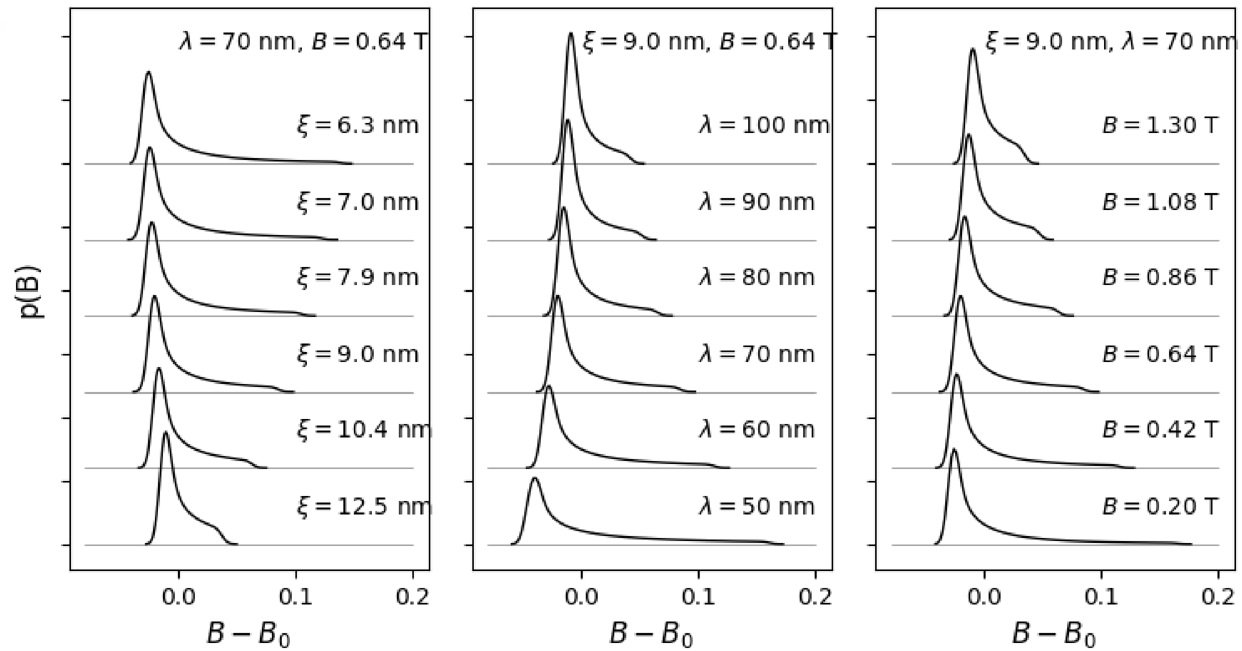
Textbook case: Nb single crystal





$$\int p(B) e^{(B'-B)^2/2\sigma^2} dB$$ Gaussian convolution

By eye: very correlated fits!
No chance of fitting λ, ξ, σ, B from single $p(B)$

Solution: global fits

muons in sample holder

Herlach et al. Hyperfine Interactions 63, 41

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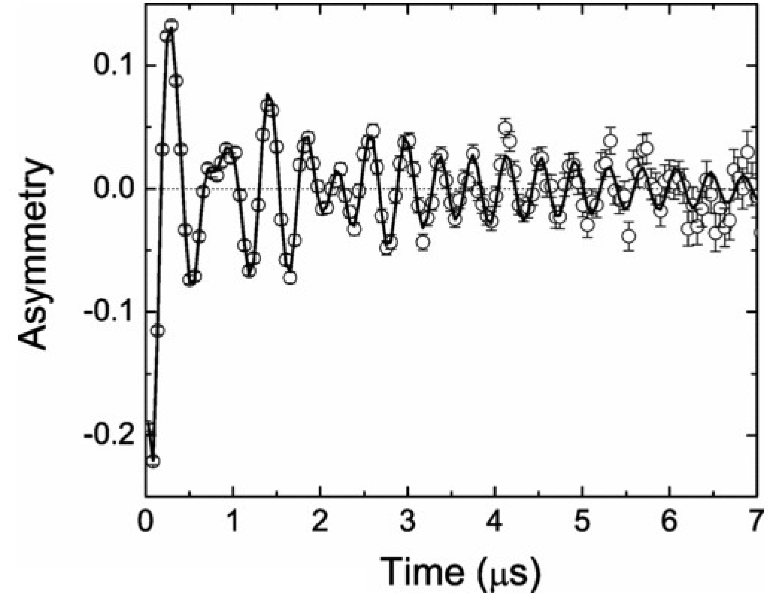
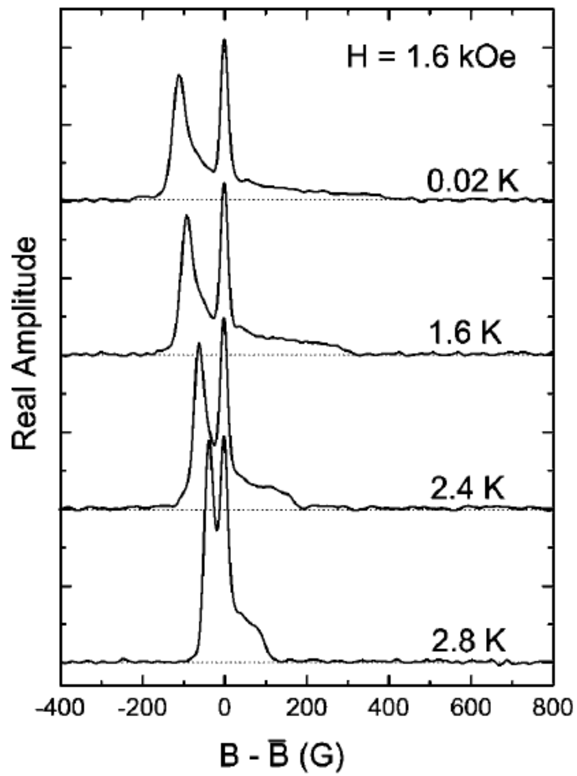
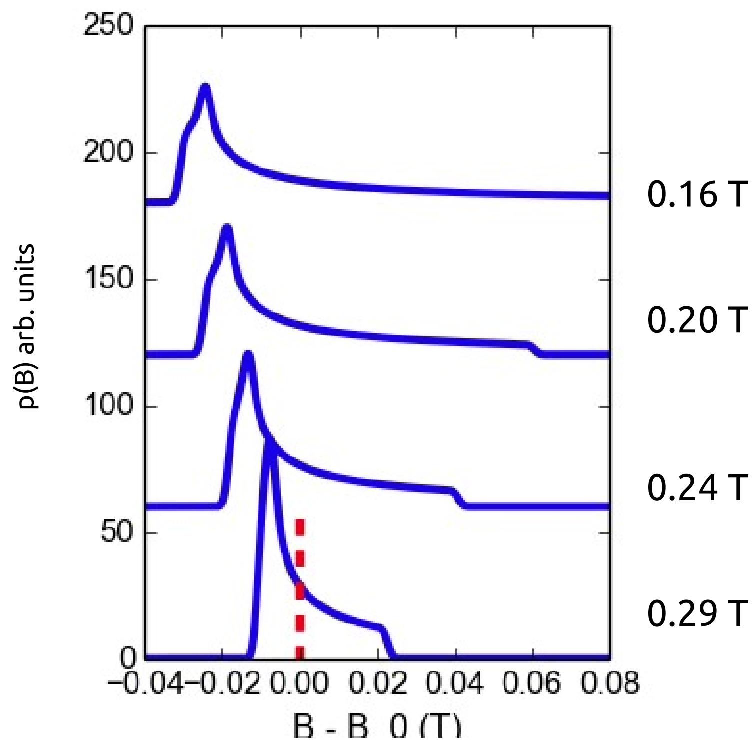
Another textbook case: Vanadium



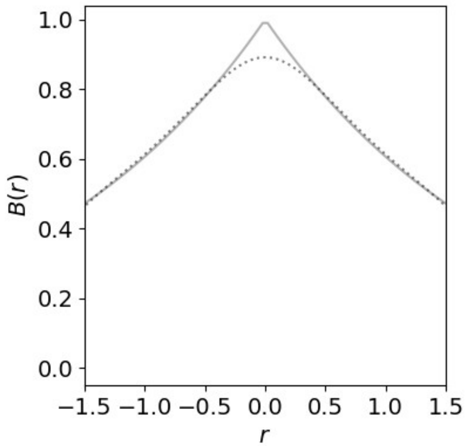
M. Laulajainen PHYSICAL REVIEW B 74, 054511 (2006)





Details – II : cut-off functions

$$B(\mathbf{r}) = \langle B \rangle \sum_{\mathbf{Q}} b(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}}$$



A *cut-off* function represents
a finite ξ
(several approximations quoted
In the literature)

$$b(\mathbf{Q}) = \frac{e^{-\xi^2 Q^2/2}}{1+\lambda^2 Q^2}$$

Actually London model $\nabla^2\mathbf{B} - \frac{1}{\lambda^2}\mathbf{B} = \Phi_0 \hat{z} \sum_{\mathbf{R}} \delta(\mathbf{R})$ approximates GL equations.

Better GL approximation for *extreme type II*
e.g. Yaouanc et al. Phys. Rev. B 55, 11107

$$b(\mathbf{Q}) = \frac{uK_1(u)(1-x^4)}{\lambda^2 Q^2}$$

K_1 modified Bessel function

$$x = \frac{B}{B_{c2}}$$

$$u = \xi Q \left(\sqrt{2} - \frac{0.75}{\kappa}\right)\sqrt{1+x^4}\sqrt{1-2x(1-x)^2}$$

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Details – I : extreme type II

$$B(\mathbf{r}) = \langle B\rangle \sum_{\mathbf{Q}} b(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}}$$

For many high T_c superconductors $\kappa = \frac{\lambda}{\xi} \geq 10$

$$b(\mathbf{Q}) = \frac{1}{1+\lambda^2 Q^2}$$

with $x = \frac{B}{B_{c2}}$ we get $\lambda^2 Q_0^2 \approx 5.4 x \kappa^2 \gg 1$

$$b(\mathbf{Q}) \propto \frac{1}{\lambda^2}$$

In this case TF-µSR provides a simple direct measurement of λ

$$\langle B^2\rangle - \langle B\rangle^2 = \langle B\rangle^2 \sum_{\mathbf{Q}\neq 0} b^2(\mathbf{Q}) \propto \frac{1}{\lambda^4}$$

the standard deviation of the µSR lineshape is proportional to λ^{-2}

$$\sigma[\mu\mathrm{s}^{-1}] = \frac{7.904 \cdot 10^4}{\lambda^2[\mathrm{nm}]}$$

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Summary

$$\int p(B) e^{(B'-B)^2/2\sigma^2} dB$$

$$B(\mathbf{r}) = \langle B \rangle \sum_{\mathbf{Q}} \frac{C(\xi Q)}{1+\lambda^2 Q^2} e^{i\mathbf{Q}\cdot\mathbf{r}}$$

parameters

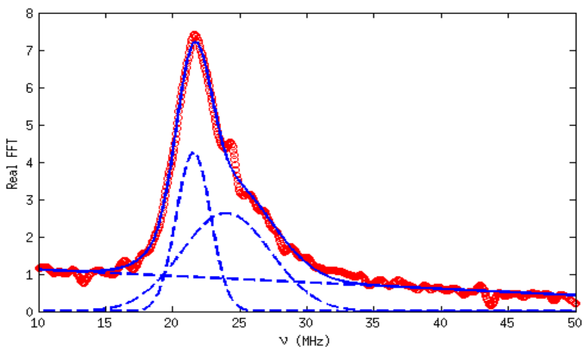
- σ inhomogeneity (disorder)
- ξ physically interesting
- λ
- C cut-off function

Sometimes it is easier to determine

$$\langle \Delta B^2 \rangle = \langle B \rangle \sum_{\mathbf{Q}\neq 0} b^2(\mathbf{Q}) \propto \frac{1}{\lambda^4}$$

extreme type II

$$\sigma = \gamma_\mu \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2}$$



from a multi-Gaussian fit

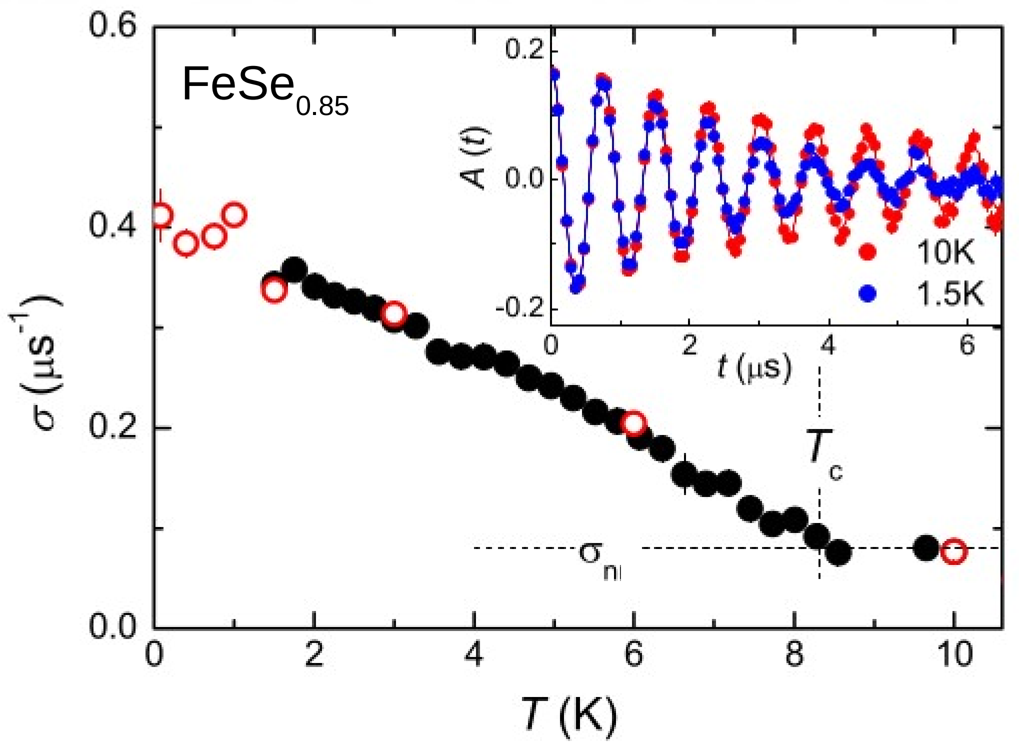
Caution: nuclear dipoles

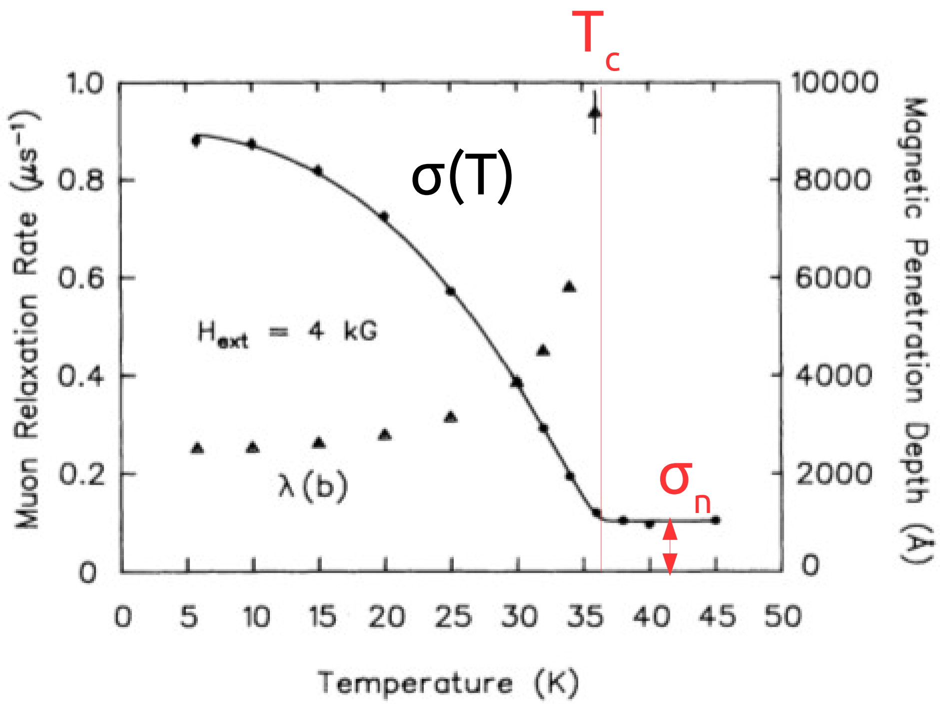
Khasanov Phys Rev B 78 220510(R)

Subtract nuclear width in quadrature!

$$\sigma_{FL} = \gamma_\mu \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2}$$

$$\sigma^2 = \sigma_{FL}^2 + \sigma_n^2$$





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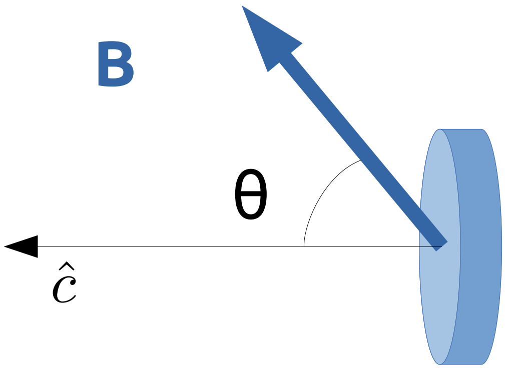
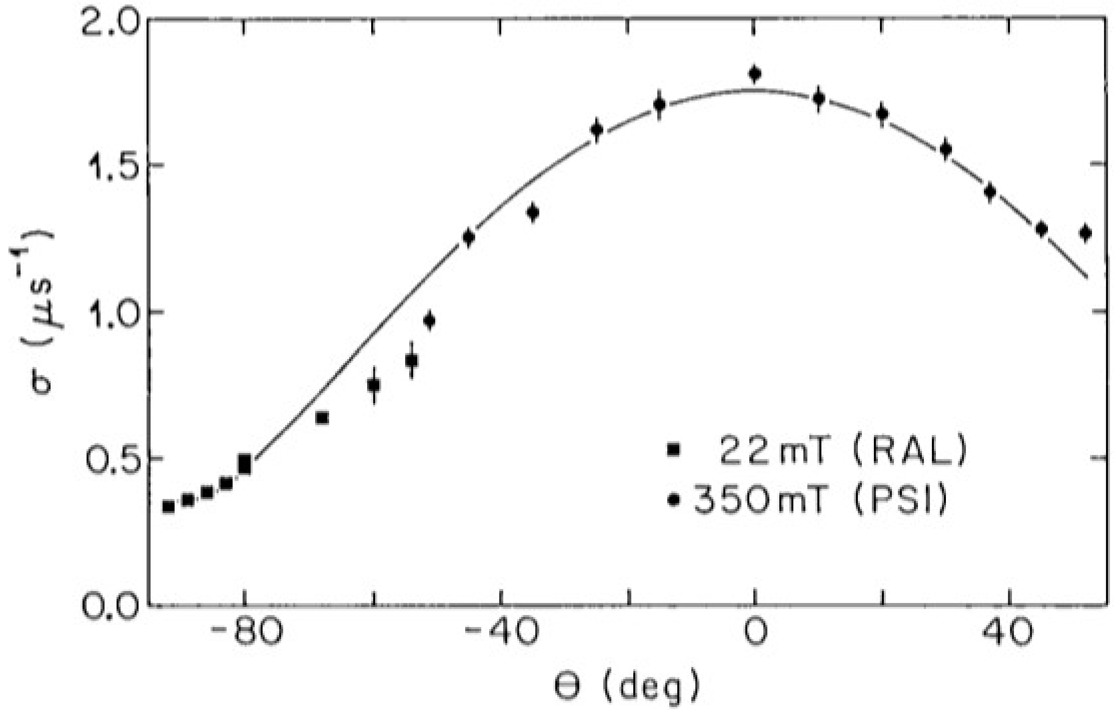
Angle dependence in $YBa_2Cu_3O_{6.9}$





for uniaxial systems

$$\lambda^2 = \frac{m}{\mu_0 n e^2} \rightarrow \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_c \end{bmatrix} \frac{1}{\mu_0 n e^2}$$

$$\sigma_{FL}(\theta) = \sigma_{FL}(0)\left[\cos^2\theta + \left(\frac{m_a}{m_c}\right)\sin^2\theta\right]^{\frac{1}{2}}$$

Forgan Hyperfine Interactions 63 41
Cubitt Physica C 213 126

Thiemann Phys Rev B 39 11406

Anisotropic polycrystal

Anisotropy dictated by

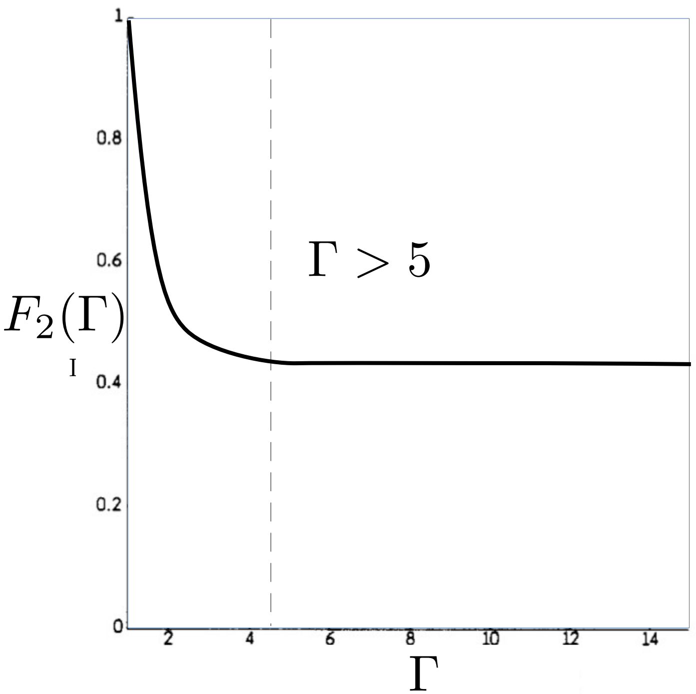
$$[\lambda^2] = \frac{1}{\mu_0 n e^2}\begin{bmatrix} m_a & 0 & 0 \\ 0 & m_a & 0 \\ 0 & 0 & m_c \end{bmatrix} \qquad \Gamma = \sqrt{\frac{m_a}{m_c}}$$

Polycrystal average

$\kappa > 10$ (London limit)

$$\sigma[\mu s^{-1}] = \frac{7.904 \cdot 10^4}{\lambda_{\text{eff}}^2[\text{nm}]}$$

$$\lambda_a = \frac{\lambda_{\text{eff}}}{\sqrt[4]{F_2}} \to 1.228\lambda_{\text{eff}}$$



Barford Physica C 156 515

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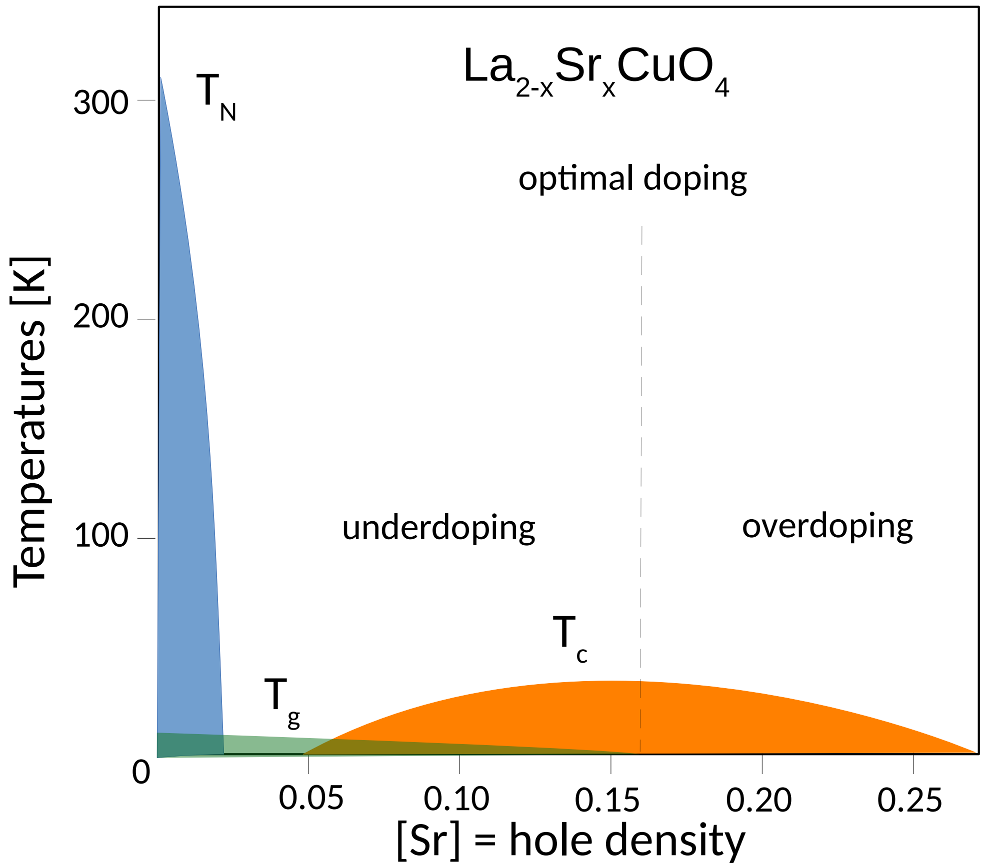
Cuprates: a flash picture



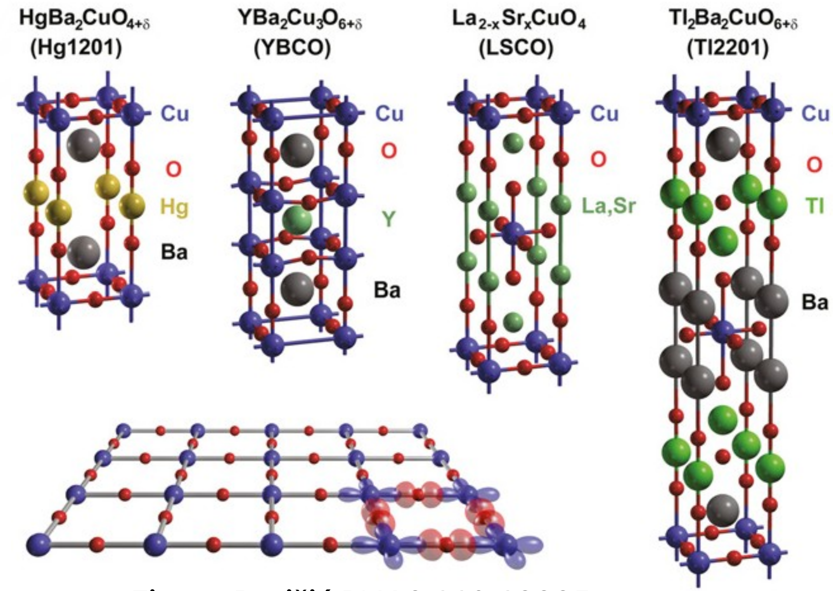


Figure Barišić PNAS 110 12235

$La^{3+}_{2-x}Sr^{2+}_{x}$ CuO_4

xh^{1+}

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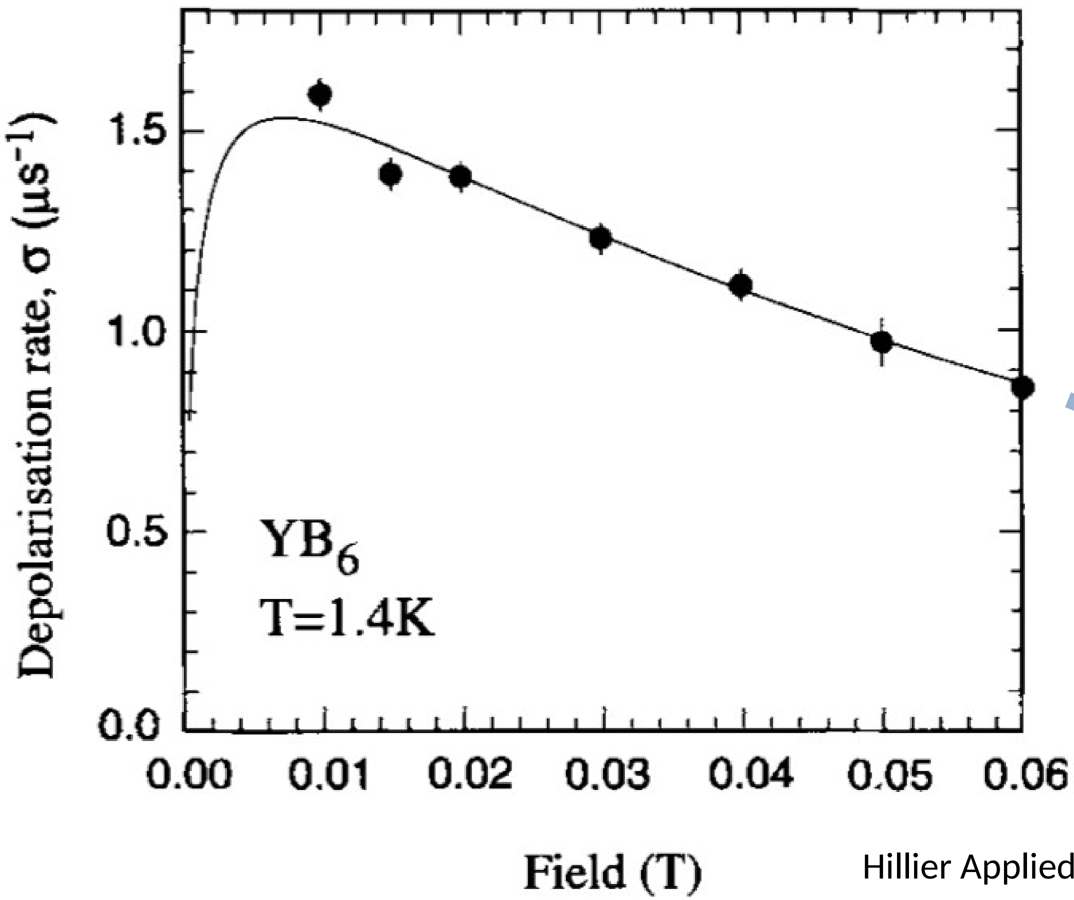
Indirect fit of coherence length

$$\sigma = \gamma_\mu \sqrt{\langle \Delta B^2 \rangle} = \gamma_\mu \langle B \rangle \sqrt{\sum_{\mathbf{Q}\neq 0} \frac{e^{-\xi^2 Q^2}}{1 + Q^2\lambda^2/(1-\frac{B}{B_{c2}})}}$$

$$Q(B) = \left(\frac{2\pi}{\alpha}\right)^2 \frac{B}{\Phi_0}$$

From field dependence



B_{c2}

Hillier Applied magnetic Resonance 13 95

Temperature dependence of σ

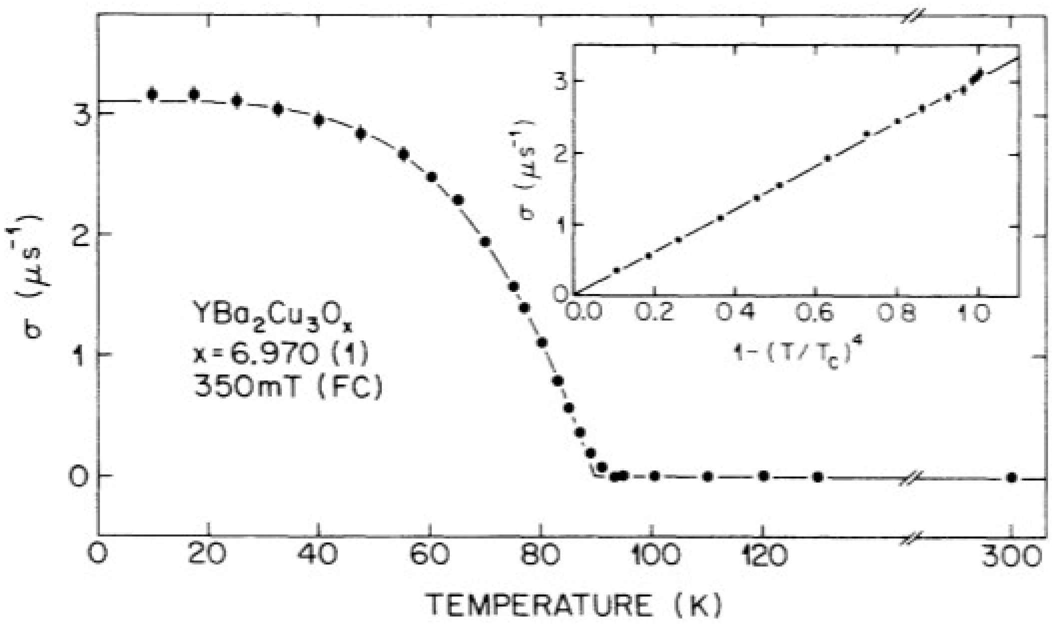
$$\frac{1}{\lambda^2} = \frac{\mu_0 e^2 n_s}{m}$$

Two fluid model, for $t = \frac{T}{T_c}$

$$\rho = \underbrace{t^4}_{\rho_n} + \underbrace{1 - t^4}_{\rho_s}$$

$$\frac{\sigma(T)}{\sigma(0)} = \rho_s(T) = \frac{\lambda^2(0)}{\lambda^2(T)}$$

normalised supercarrier density



$$\rho_s(t) = 1 - t^4$$

Caution
- polycrystals
- missing low temperature

Pümpin Phys Rev B 42 8019

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A glimpse on BCS

Two main ingredients for BCS

- Attractive *V* between electrons, mediated by a boson

$$\chi_{e-ph}(\omega) = \frac{g^2_{e-ph}}{\cancel{\omega^2} - \omega_D^2}$$

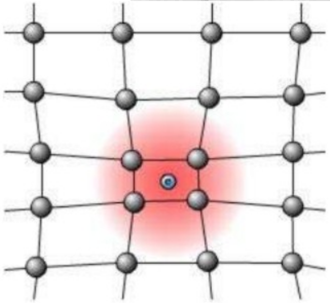
Weak coupling, adiabatic* limit, a hierarchy of energies $\hbar\omega \approx k_B T \ll \hbar\omega_D \ll \epsilon_F$

- Presence of a Fermi sea

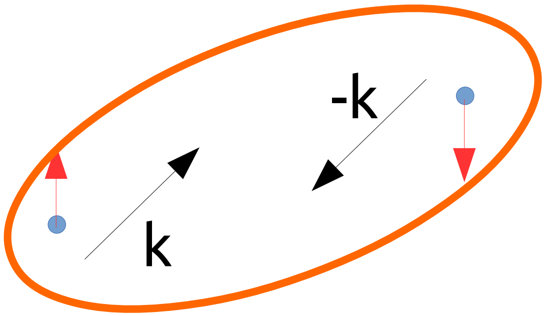
Fermi liquid screening

$$V = \underbrace{(\cancel{1} + \chi)}_{\epsilon(\omega)} \frac{e^2}{r} \approx \chi \frac{e^2}{r} < 0$$

Phonon-mediated attraction



→ Cooper pairs



* Migdal theorem

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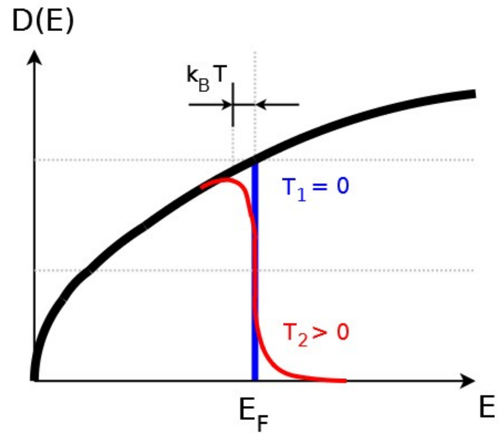
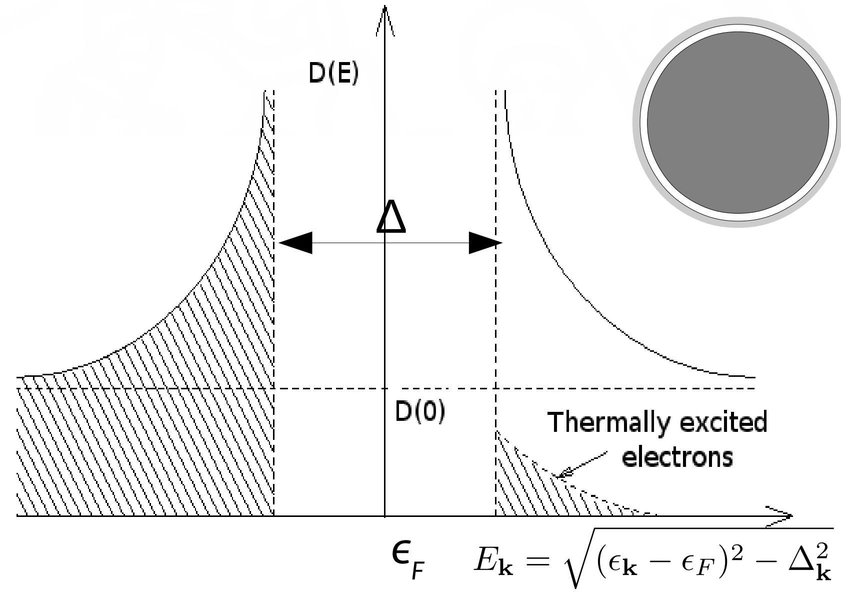
BCS gap

normal metal
DOS

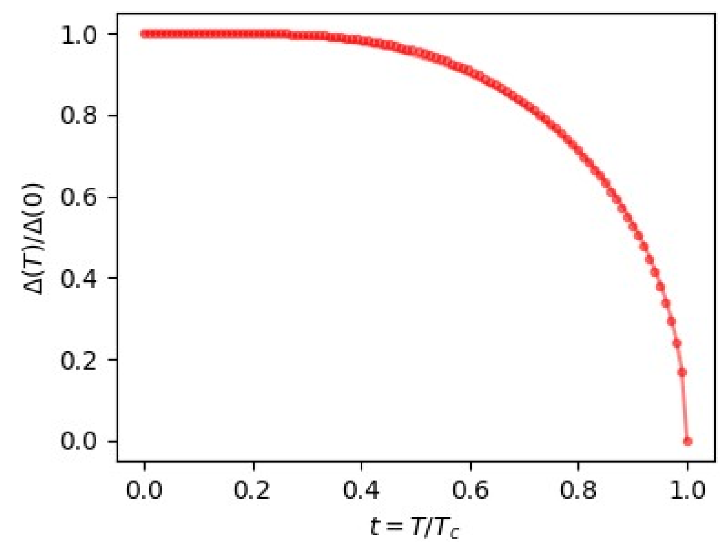




below T_c gap opens in DOS at ϵ_F

At T = 0 $\quad \Delta(0) = 1.764\, k_B T_c$ and the temperature dependence is

$$\Delta(T) = \Delta(0) \tanh\left(\frac{\pi T_c}{\Delta(0)}\sqrt{\frac{1-t}{t}}\right)$$



new definition of coherence length $\quad \xi_0 = \dfrac{\hbar v_F}{\pi \Delta(0)}$

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ρ(t) beyond two fluid model

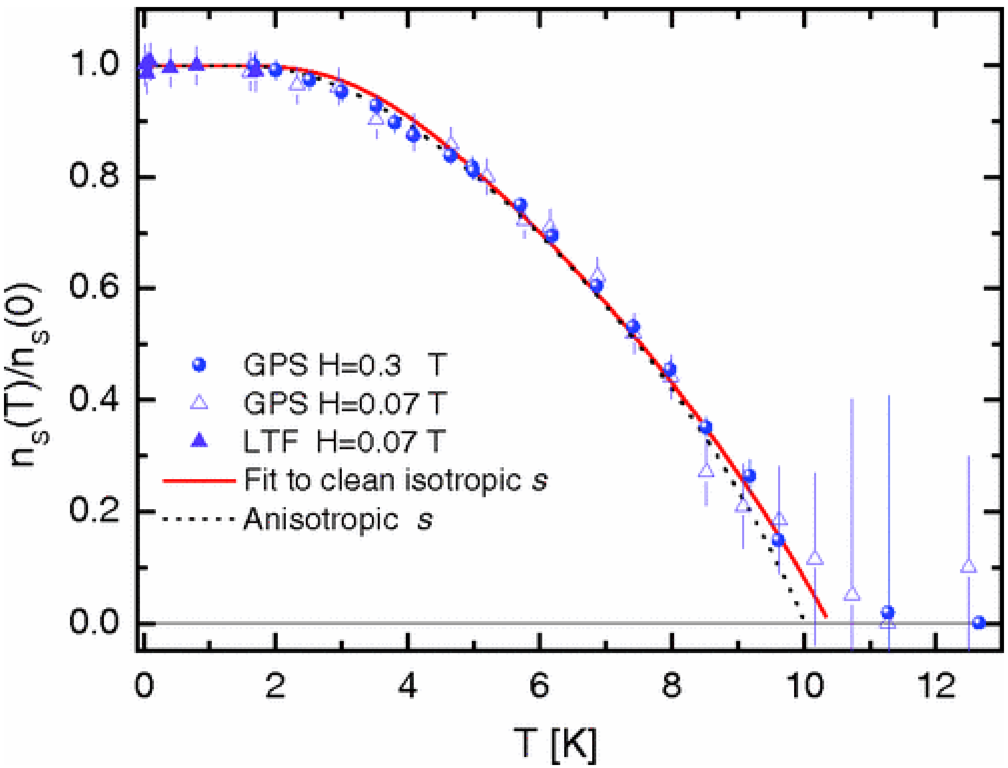
$$t = \frac{T}{T_c}$$

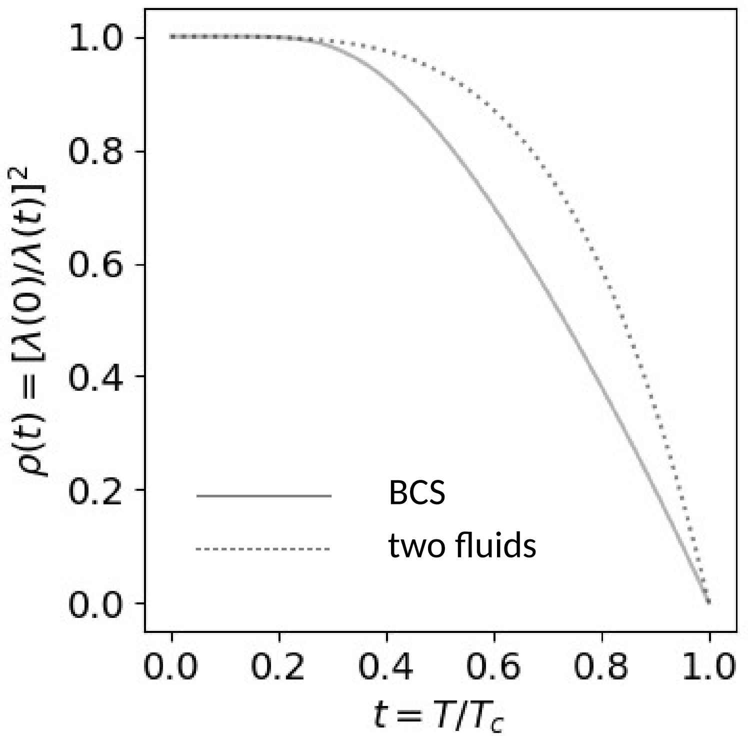
calculate current response function

BCS supercarrier density

$$\rho(t) = \frac{n_s(T)}{n_s(0)} = 1 - \frac{1}{2t}\int_0^\infty \frac{dx}{\cosh^2\left[\frac{\sqrt{x^2+\delta^2(t)}}{2t}\right]}$$

e.g. $LaO_{0.5}F_{0.5}BiS_2$





Lamura Phys Rev B 88, 180509(R)

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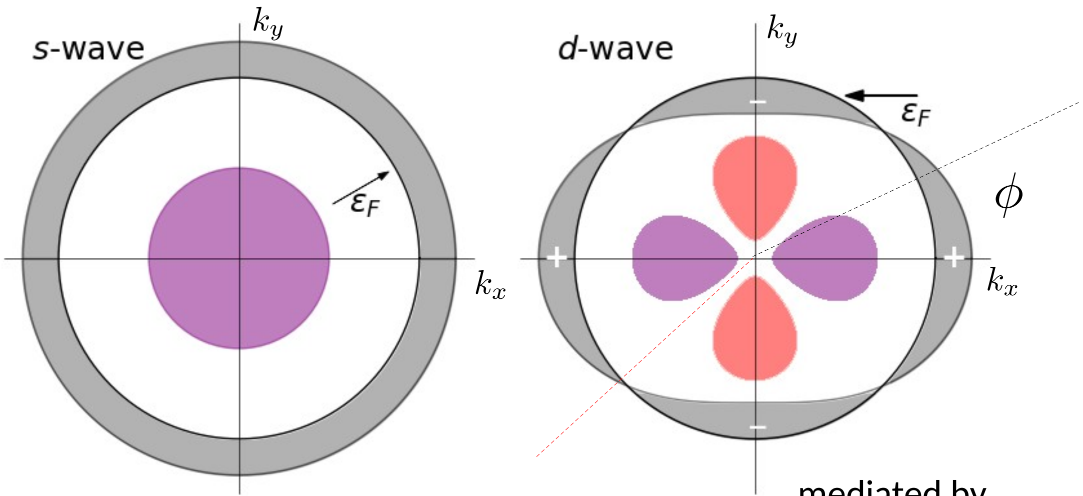
Beyond phonon coupling

BCS weak coupling assumes $\Delta_{\mathbf{k}} = \Delta$ constant in **k** space

Other couplings may have different symmetries

$$\Delta(\hat{k}) = \Delta(0) g(\hat{k})$$

symmetry	$\Delta(0)$	$g(\hat{k})$
s	$1.763\, k_B T_c$	1
d	$2.14\, k_B T_c$	$\cos 2\phi$



lines of nodes

mediated by spin fluctuations in cuprates

$$\Delta(\phi) = 0 \qquad \phi = \frac{\pi}{4}(2n+1)$$

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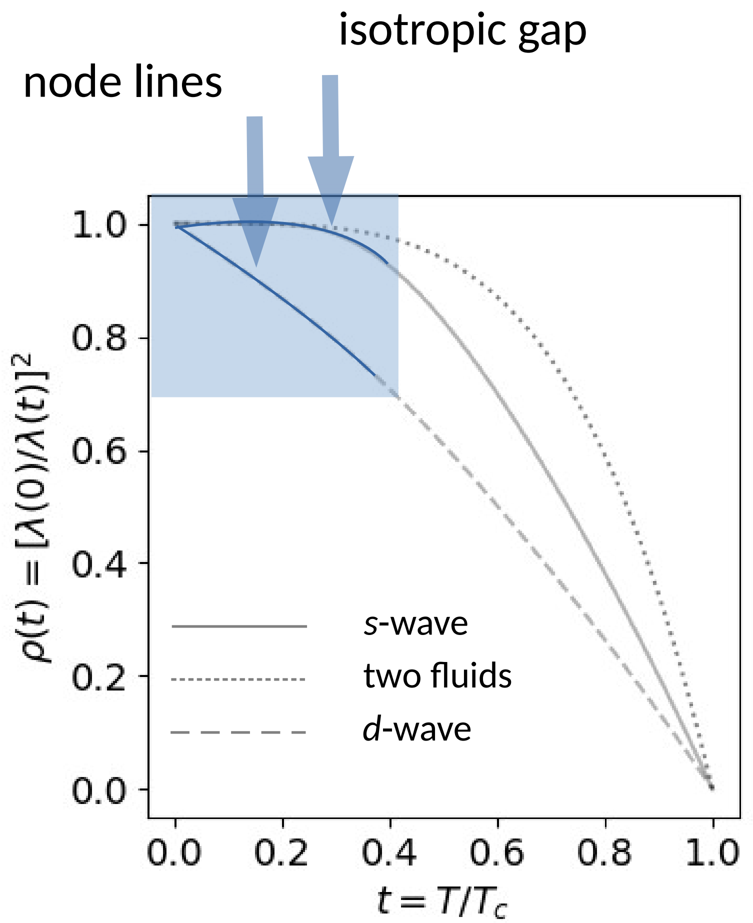
BCS gap: different coupling

A useful equation for the gap temperature dependence
Gross Zeitschrift fur Physik B: Condensed Matter 82, 243

$$\Delta(T) = \Delta(0)\tanh\left(\frac{\pi T_c}{\Delta(0)}\sqrt{\alpha\frac{1-t}{t}}\right)$$

symmetry	$\Delta(0)$	$g(\hat{k})$	α
s	$1.763\,k_B T_c$	1	1
d	$2.14\,k_B T_c$	$\cos 2\phi$	$\frac{4}{3}$



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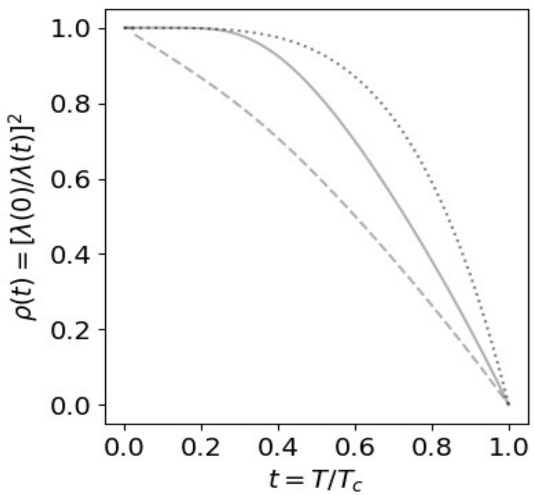
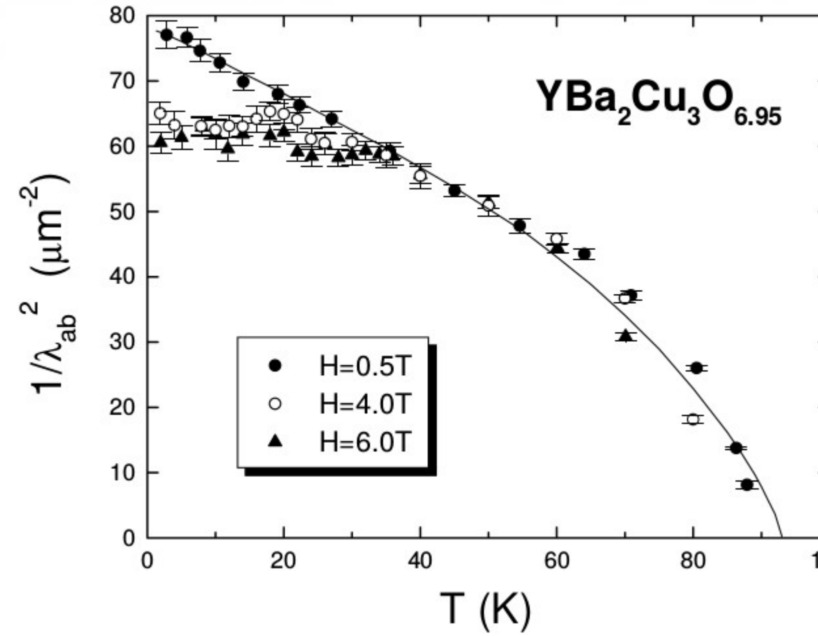
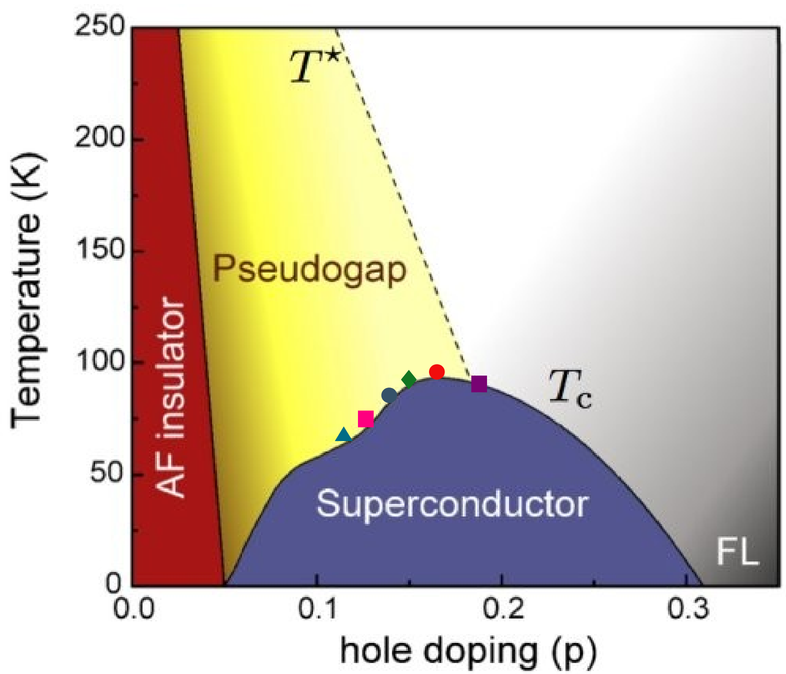
d-wave gap in cuprates



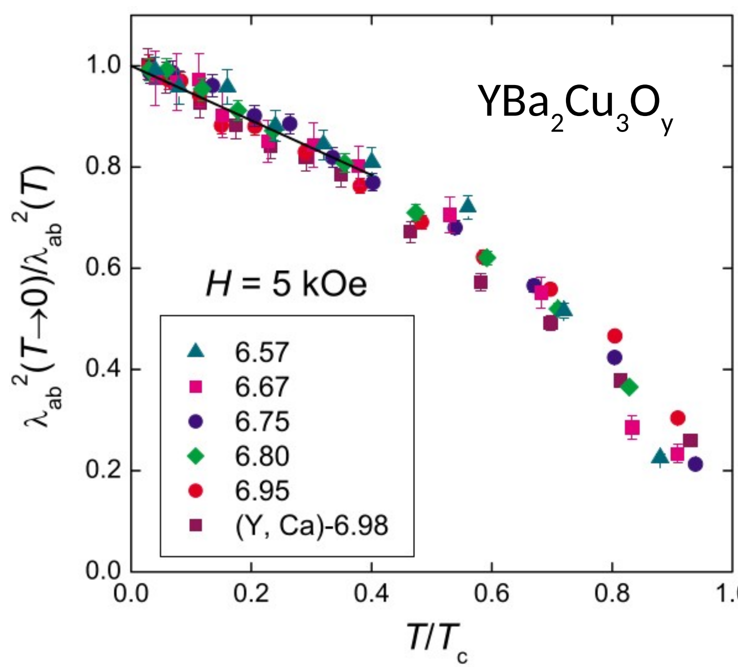
Linear T dependence due to quasi-particles (e^-) excited across the gap

Sonier Phys Rev Lett 83 4156





From LNCMI Toulouse web page



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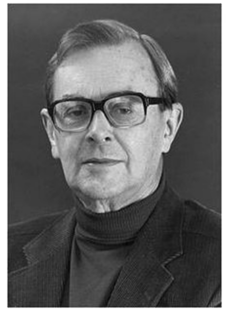
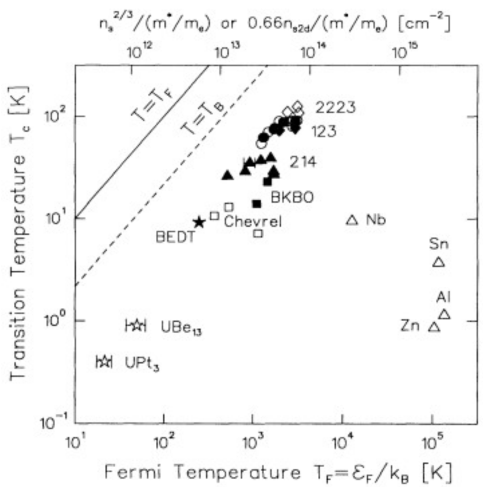
Next lesson

Dirty superconductors



A phase diagram for superconductivity



Commonalities

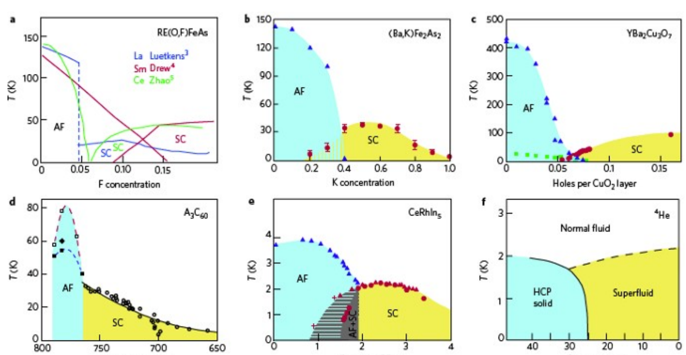


Fig. 11.1 Examples of phase diagrams showing commonalities of unconventional superconducting compound [Uem09] c, YBaCuO6+x, is from [San04]

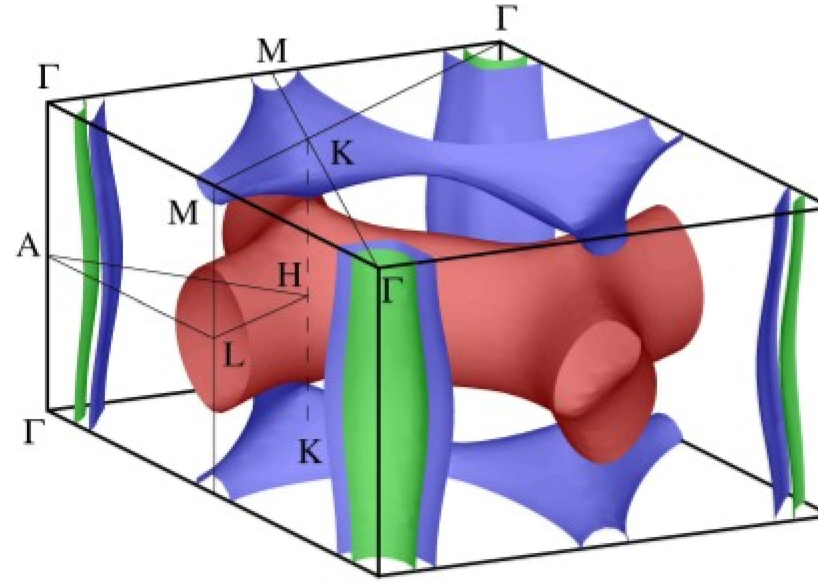
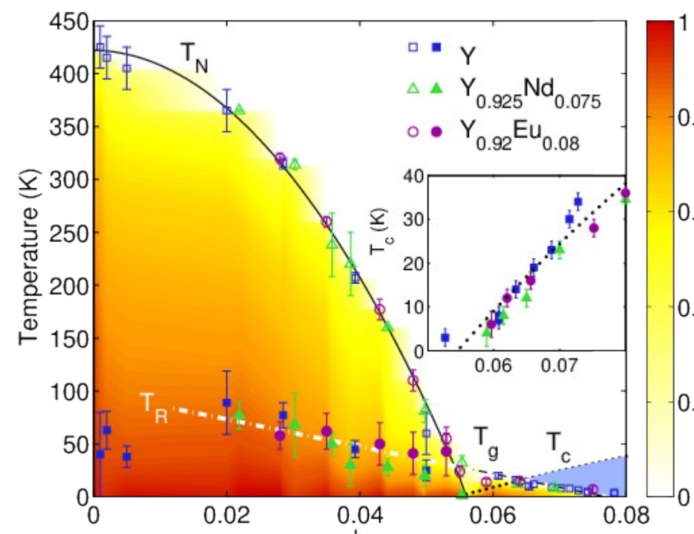


FIG. 3. Fermi surface of MgB_2.

Multi gap superconductivity



Nanoscopic coexistence

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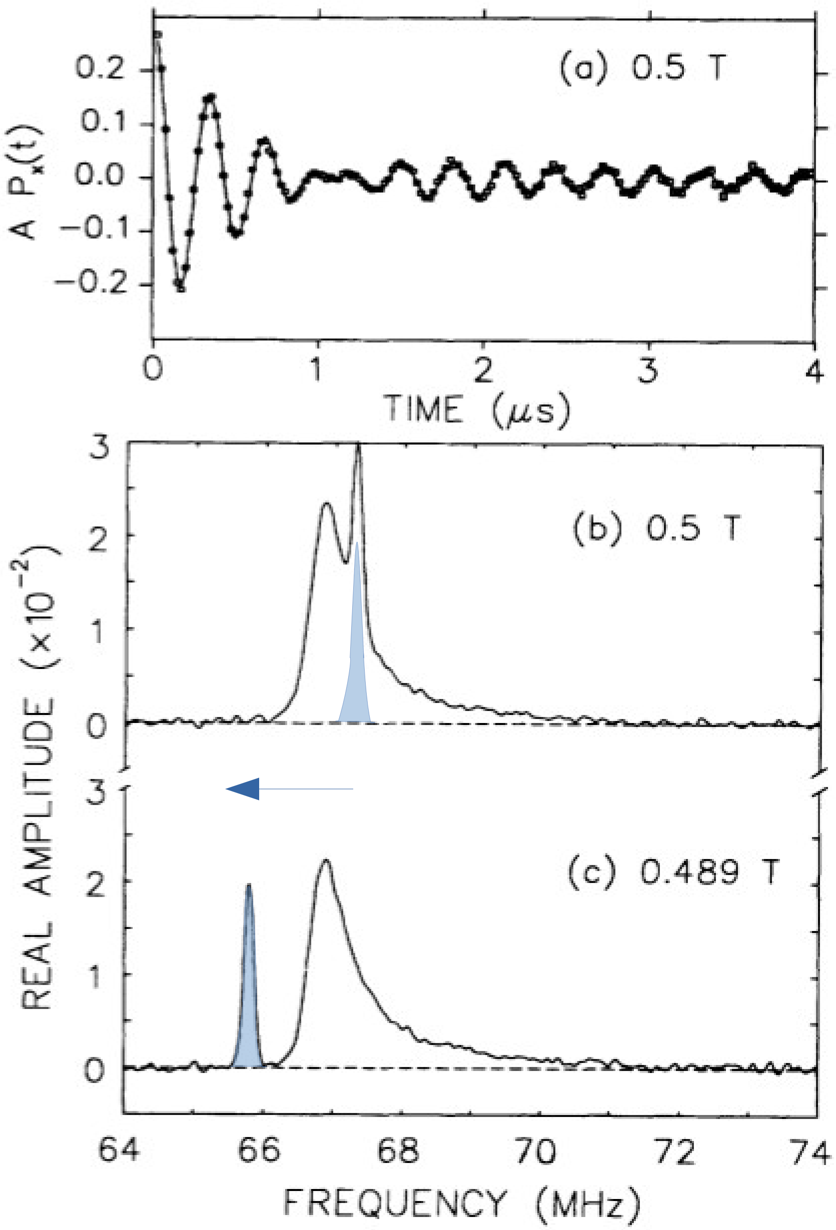
Flux pinning in $YBa_2Cu_3O_{6.95}$ crystals

(by Hardy Bonn and Liang)



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Field cooling to 5.4 K in 0.5 T

Lowering the field at 5.4K by 11 mT
only muons in sample holder shift

The flux in the superconductor is trapped

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