Muons in superconductors

- Lesson I – the land we are exploring
  - Introduction: superconductivity, a story of three length-scales
  - London equations and the penetration depth
  - Ginzburg Landau equations and the coherence length
- Lesson II – the workhorse of μSR
  - The Abrikosov flux lattice
  - Muon determination of the penetration depth
  - Conventional and unconventional superconductivity: a glance
  - BCS: the gap and its temperature dependence
- Lesson III – material science
  - Clean vs. dirty superconductors
  - A phase diagram for superconducting materials
  - Towards atomic scale coherence: nanoscopic coexistence
  - Triplet superconductivity, topological superconductivity (?)

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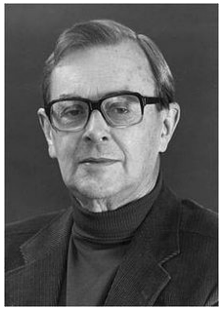
The third lengthscale

Well, wasn't it $a_{\triangle} = \sqrt[4]{\frac{4}{3}}\sqrt{\frac{\Phi_0}{B}}$?



Brian Pippard

What is the averaging length for thermodynamic fields?

depends on an intrinsic scale ξ_0

and on mean free path ℓ

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}$$

correction for impurities

Pippard *s*-wave correction on the superfluid density, for $\sigma \propto \frac{1}{\lambda^2}$

$$\sigma = \sigma(0)\left(1 + \frac{\xi_0}{\ell}\right)$$

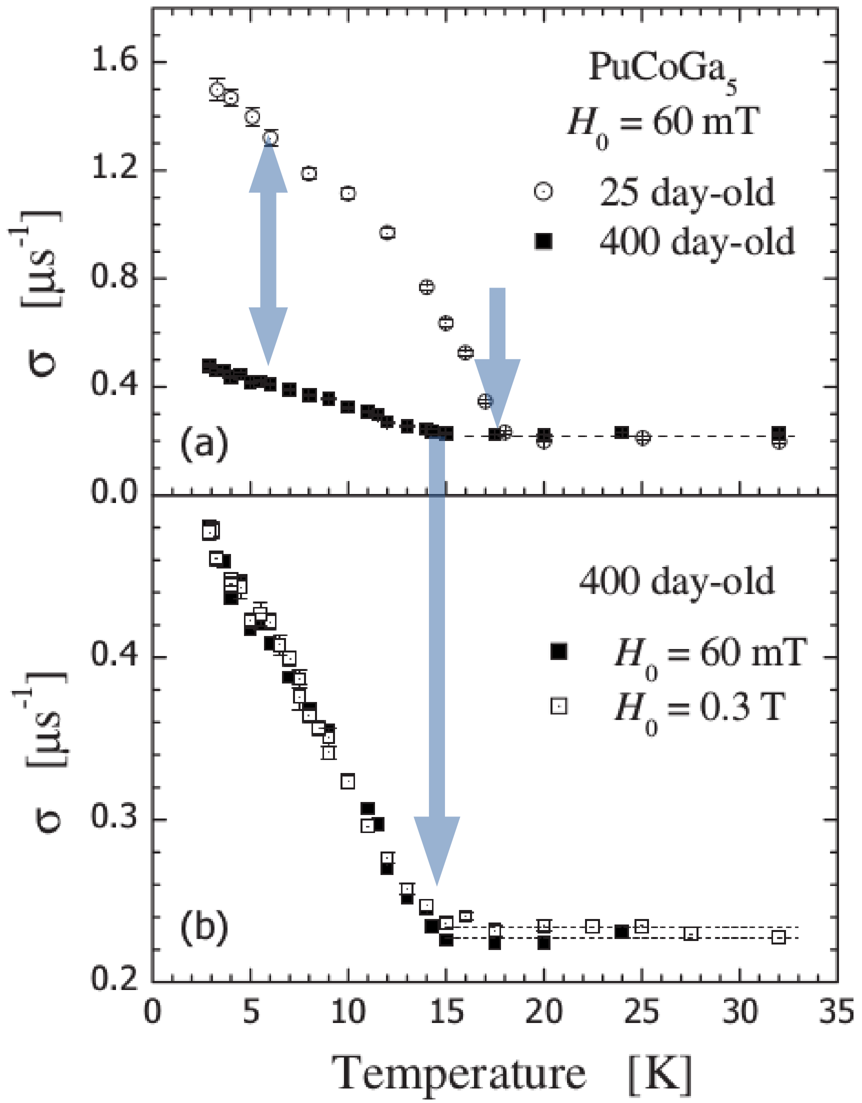
for $\xi_0 < \ell$

Good news:
many new superconductors are always in the clean limit

material	B_{c2} [T]	ξ_0 [nm]
$YBa_2Cu_3O_{6.92}$	≈ 200	≈ 1
$LaFeAsO_{0.9}F_{0.1}$	≈ 100	≈ 1.8
K_3C_{60}	50	2.8
MgB_2	40	2.9

A prototype case: $PuCoGa_5$

d-wave!



Ohishi Phys Rev B 76 064504

T_{c0} = 18.5 K $\xi_0 \approx 2\mathrm{nm}$

^{239}Pu decays in ^{235}U in 24110 yr

mean distance between defects after 400 days is $\ell \approx 20\mathrm{nm}$ (6% impurities)

$$T_c - T_{c0} \approx 3\,\mathrm{K}$$

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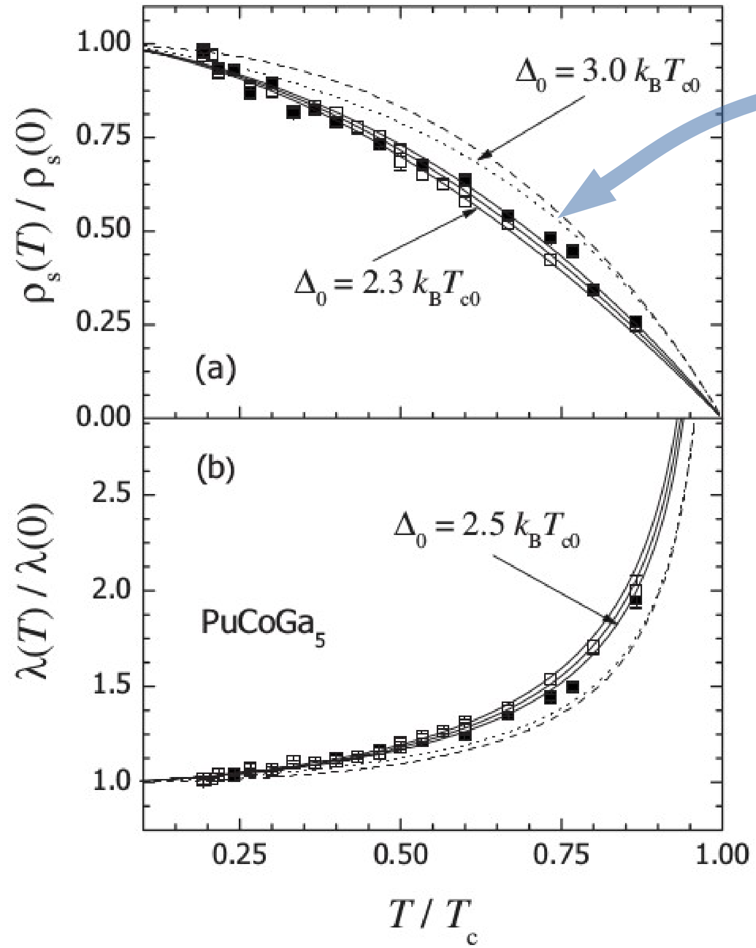
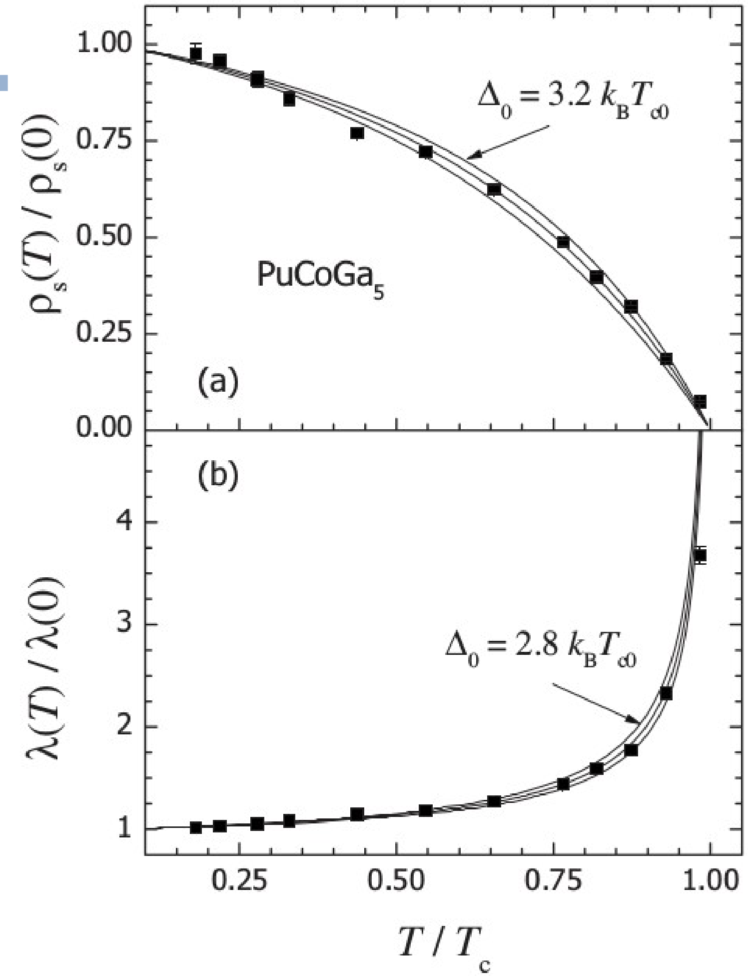
d-wave dirty limit

400 days



25 days



Ohishi Phys Rev B 76 064504

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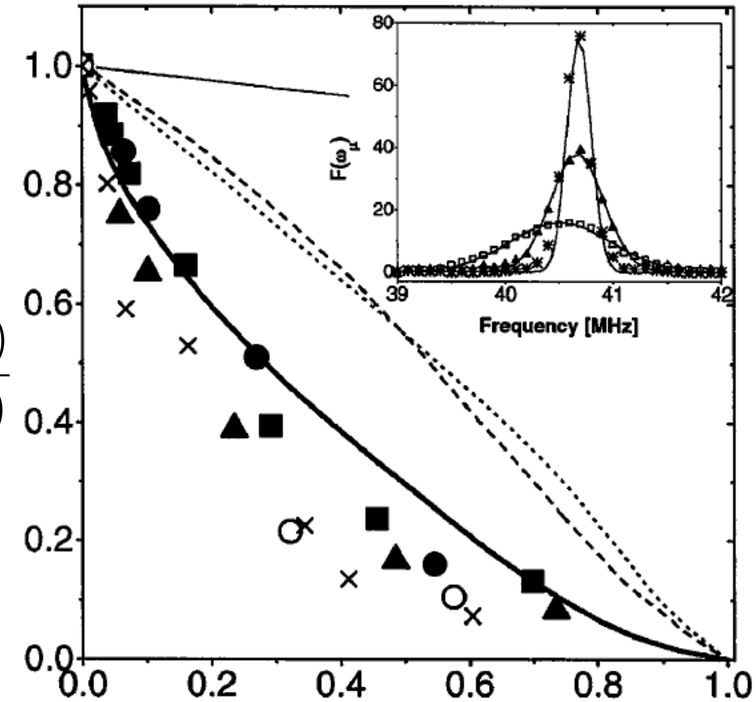
d-wave scattering Zn:$YBa_2Cu_3O_{6.9}$

$$\frac{\rho(x)}{\rho(0)} = \frac{\lambda^2(0)}{\lambda^2(x)} = \frac{\sigma(x)}{\sigma(0)}$$

- ▲ overdoped $Y_{0.8}Ca_{0.2}BaCu(Cu_{1-x}Zn_x)_2O_{6.92}$
- ■ optimally doped $Y_{0.8}Ca_{0.2}BaCu(Cu_{1-x}Zn_x)_2O_{6.62}$
- ● underdoped $Y_{0.8}Ca_{0.2}BaCu(Cu_{1-x}Zn_x)_2O_{6.62}$
- × optimally doped $YBaCu(Cu_{1-x}Zn_x)Cu_2O_{6.92}$

$\frac{\rho(x)}{\rho(0)}$



$\frac{T_c(0) - T_c(x)}{T_c(0)}$

- - - - *d*-wave Born $\ell \gg \xi_0$

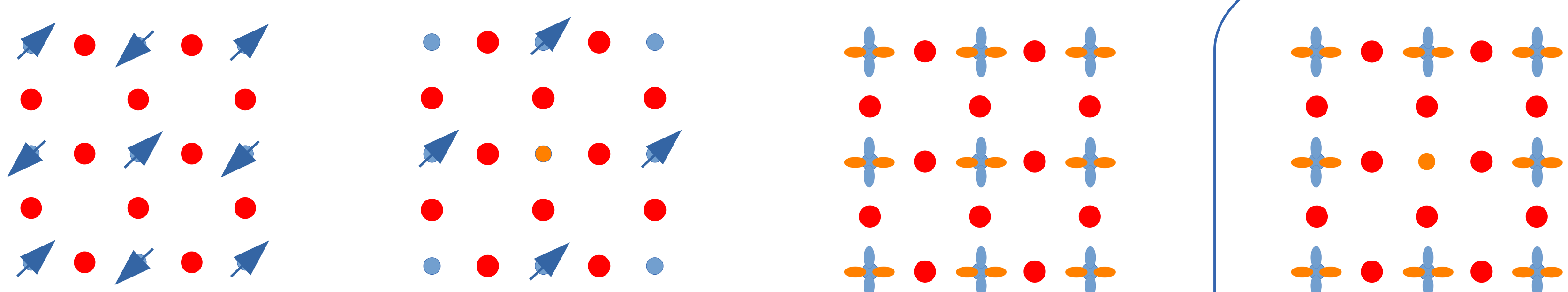
—— *s*-wave $\ell \gg \xi_0$

—— *d*-wave unitary $\ell \gg \xi_0$

······ *s*-wave $\ell \ll \xi_0$

ebvidence of d-wave pairing,
est fit by Coulomb scattering unitary limit

Bernhard Phys Rev Lett 77 2304
Bucci Physica C 235 1849



AF parent

s magnetic Zn

SC

Zn *d* non magnetic

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Beyond weak coupling

Given the cut-off boson energy $\hbar\omega_b$

Still adiabatic: $\hbar\omega_b \ll \epsilon_F$ but not limited to $k_B T_c \ll \hbar\omega_b$ like

BCS – one parameter: the *pseudopotential* μ

e.g. $$k_B T_c = \frac{\hbar\omega_b}{0.882}\exp\left(-\frac{1}{\mu}\right)$$

$$\mu = N(0)\overline{V}$$

Ignores e-e repulsion: $\overline{V} = \underbrace{\chi_{e-b}}_{<0} V_C$

Eliashberg – two parameters: the pseudopotential μ
and a realistic boson spectrum λ

e.g. $$k_B T_c = \frac{\hbar\omega_b}{1.45}\exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right)$$

$$\mu^* = \frac{\mu}{1+\mu\ln\frac{\epsilon_F}{\hbar\omega_b}}$$ corrects for e-e repulsion

λ spectral function integral

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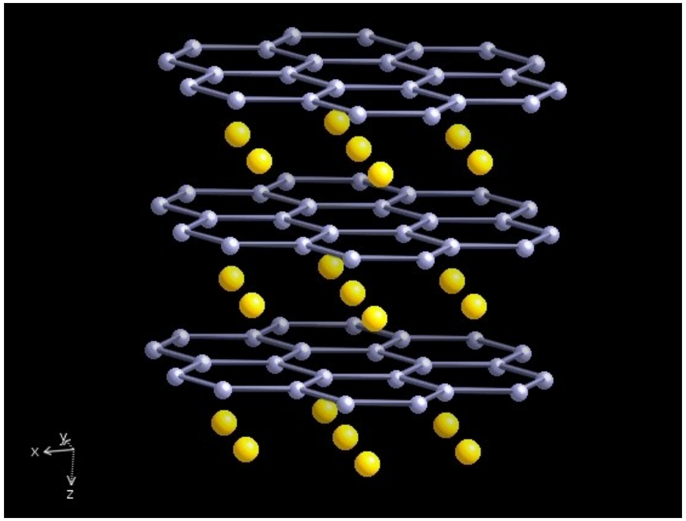
MgB_2: a tale of two gaps



3-d π bands

2-d σ bands

both cross ϵ_F

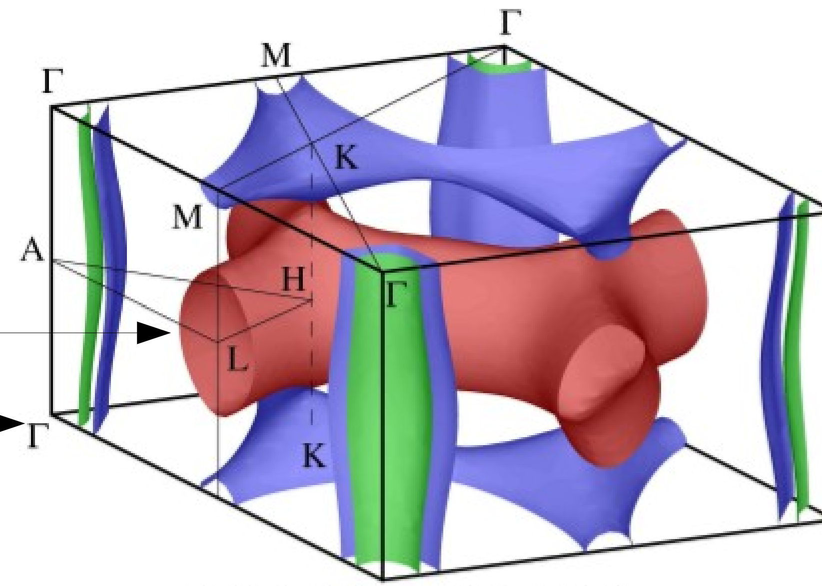


FIG. 3. Fermi surface of MgB_2.

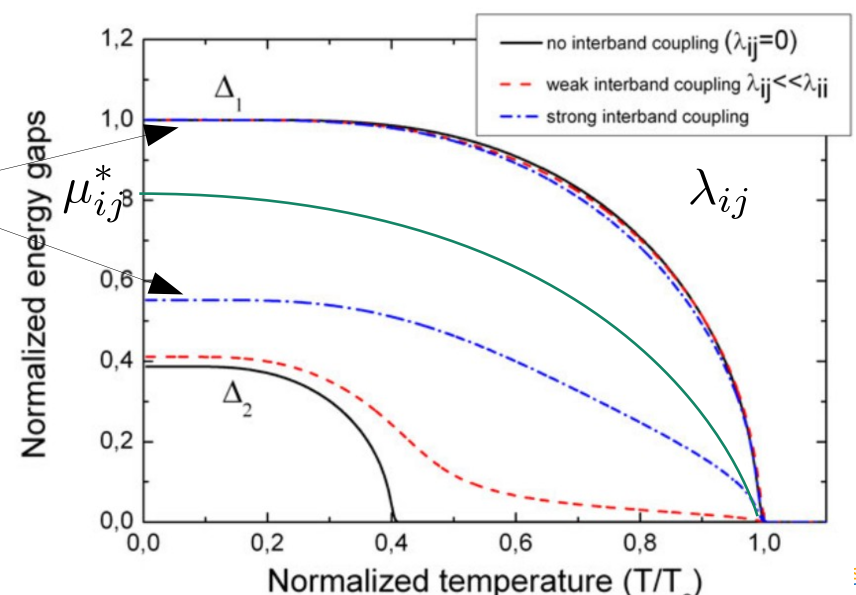
Mazin, Physica C 385 49

Can there be more gaps?

T_c = 39 K strong coupling

Eliashberg theory requires n^2 pseudopotentials μ^*_{ij} and n^2 spectral functions λ_{ij}

Two gaps with same T_c visible at intermediate λ_{ij}



Ummarino, unpublished

λ_{ii}	$\lambda_{ij}, \quad i \neq j$	
intraband	inter-band	with intraband scattering

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MgB_2 supercarrier density $\sigma(T)$

sum of two contributons

$$\sigma(t) = \sigma(0)\left[w_\sigma\, \rho_\sigma(t) + (1-w_\sigma)\rho_\pi(t)\right]$$

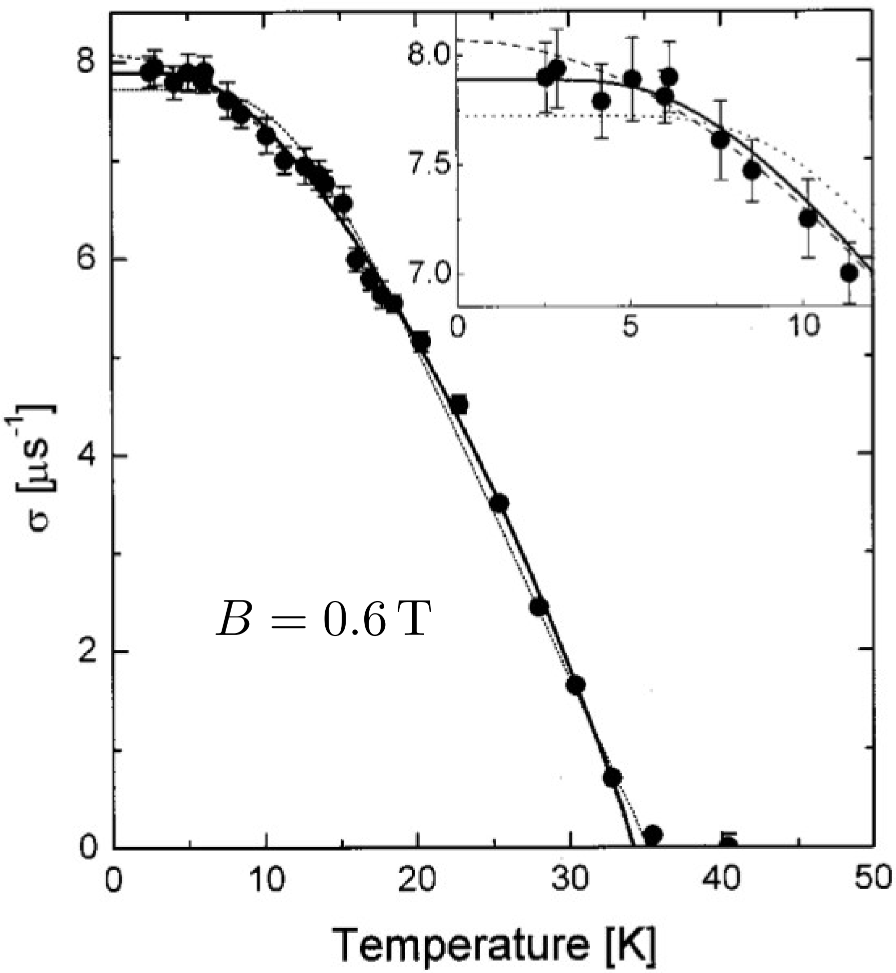
s-wave, as seen this morning

$$\rho_i(t) = \frac{n_i(t)}{n_i(0)} = 1 - \frac{1}{2t}\int_0^\infty \frac{dx}{\cosh^2\left[\frac{\sqrt{x^2+\delta_i^2(t)}}{2t}\right]}$$

$$t = \frac{T}{T_c} \qquad x = \frac{B}{B_{c2}}$$

$$\delta_i = \frac{\Delta_i(T)}{\Delta_i(0)} = \alpha_i T_c$$

$\Delta_\sigma = 6.0(3) \quad \Delta_\pi = 2.6(3)$



Niedermayer Phys Rev B 65, 094512

Polycrystal data

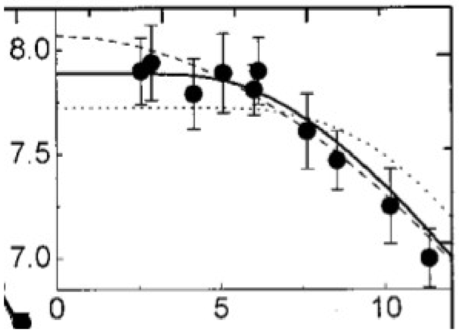
$\sigma(0) \rightarrow \lambda_a = 1.23\lambda_{\rm eff}$

Second moment vs. field

Serventi Phys Rev Lett 93, 217003

$$\sigma(t) = \sigma(0)\left[w_\sigma\,\rho_\sigma(t) + (1-w_\sigma)\rho_\pi(t)\right]$$

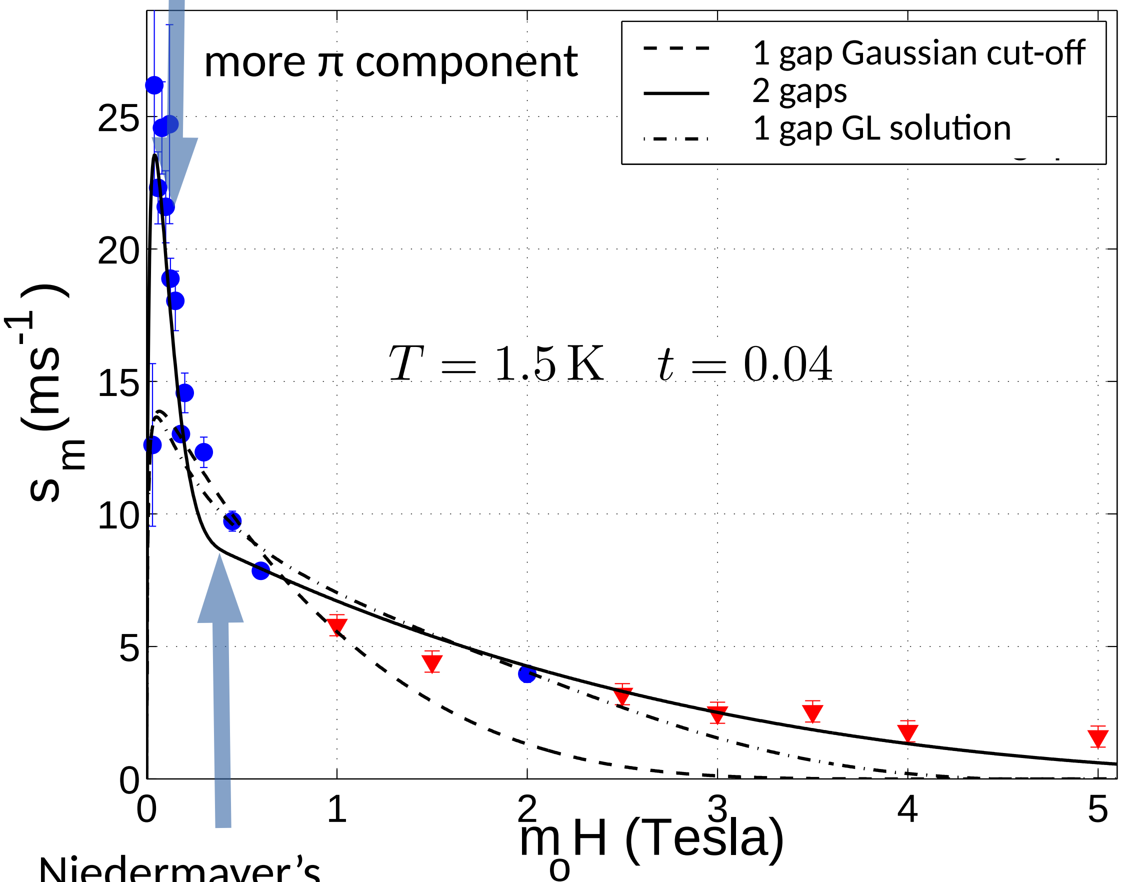
very small at B=0.6 T



$$Q(B) = \left(\frac{2\pi}{\alpha}\right)^2 \frac{B}{\Phi_0}$$

$$\sigma(B) = \gamma_\mu \left[\sum_{\mathbf{Q}\neq 0} w_\sigma\, b_Q^2(\xi_\sigma,\lambda) + (1-w_\sigma) b_Q^2(\xi_\pi,\lambda)\right]^{\frac{1}{2}}$$

$$\xi_\sigma = 5.0\,\mathrm{nm}, \quad \xi_\pi = 25(2)\,\mathrm{nm}$$



Niedermayer's experiment

$$b_Q(\xi,\lambda) = \frac{\exp\left(-\frac{\xi^2 Q^2}{1-Q_0\xi/\sqrt{3\pi}}\right)}{1+\frac{\lambda^2 Q^2}{1-Q_0\xi/\sqrt{3\pi}}}$$

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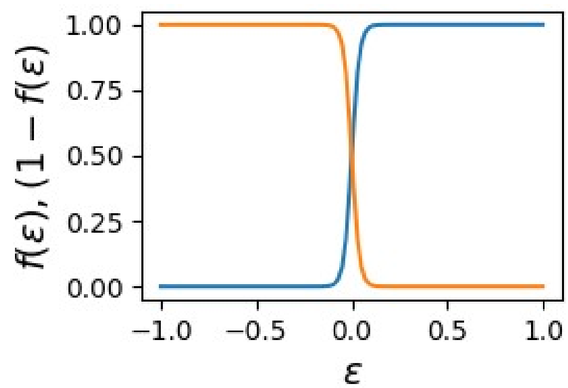
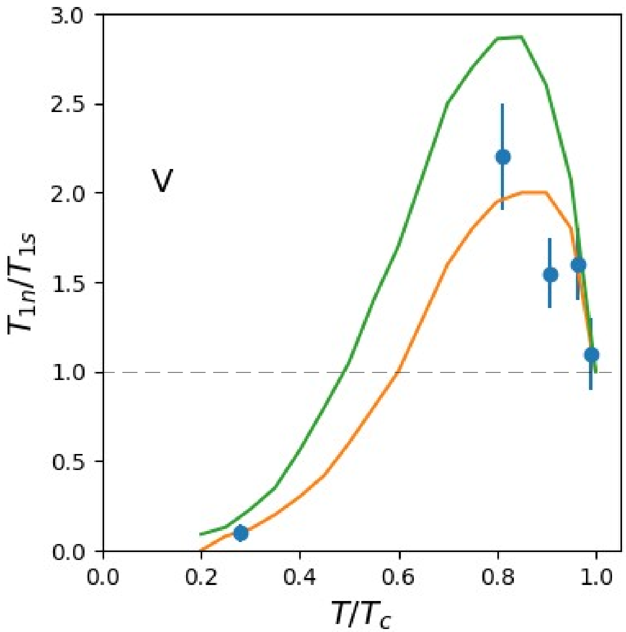
The Hebel-Slichter coherence peak

$$\frac{1}{T_{1n}} = -\frac{\pi k_B T}{\hbar} A_0^2 \int_{-\epsilon_F}^{\infty} N^2(\epsilon) \frac{\partial f(\epsilon)}{\partial \epsilon} d\epsilon$$
$$= \frac{\pi k_B T}{\hbar} A_0^2 N^2(0)$$

Nuclear relaxation in metals (Korringa)





data from Hebel and Slichter Phys. Rev. 107, 901 (**1957**)

The green and orange curves (upper and lower bounds) are:

$$E = \sqrt{\epsilon^2 + \Delta^2}$$

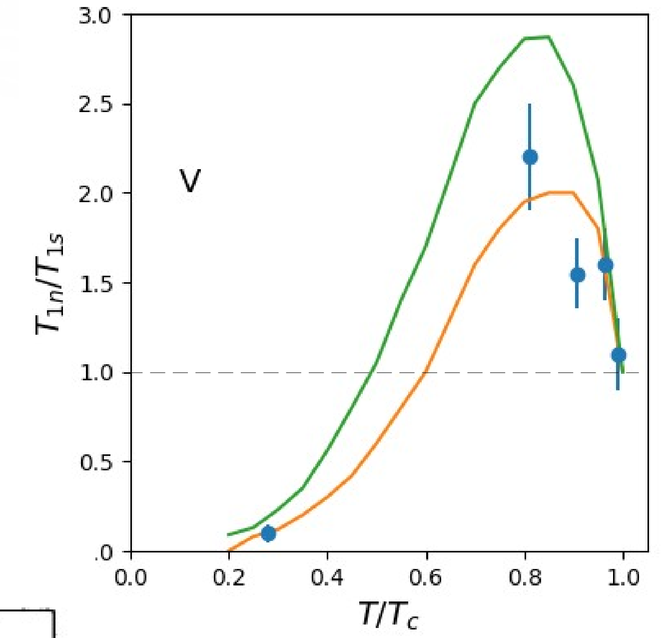
$$\frac{1}{T_{1s}} = -\frac{\pi k_B T}{\hbar} A_0^2 \int_{-\epsilon_F}^{\infty} N^2(E) \frac{\partial f(E)}{\partial \epsilon} C(\epsilon, \Delta) d\epsilon$$

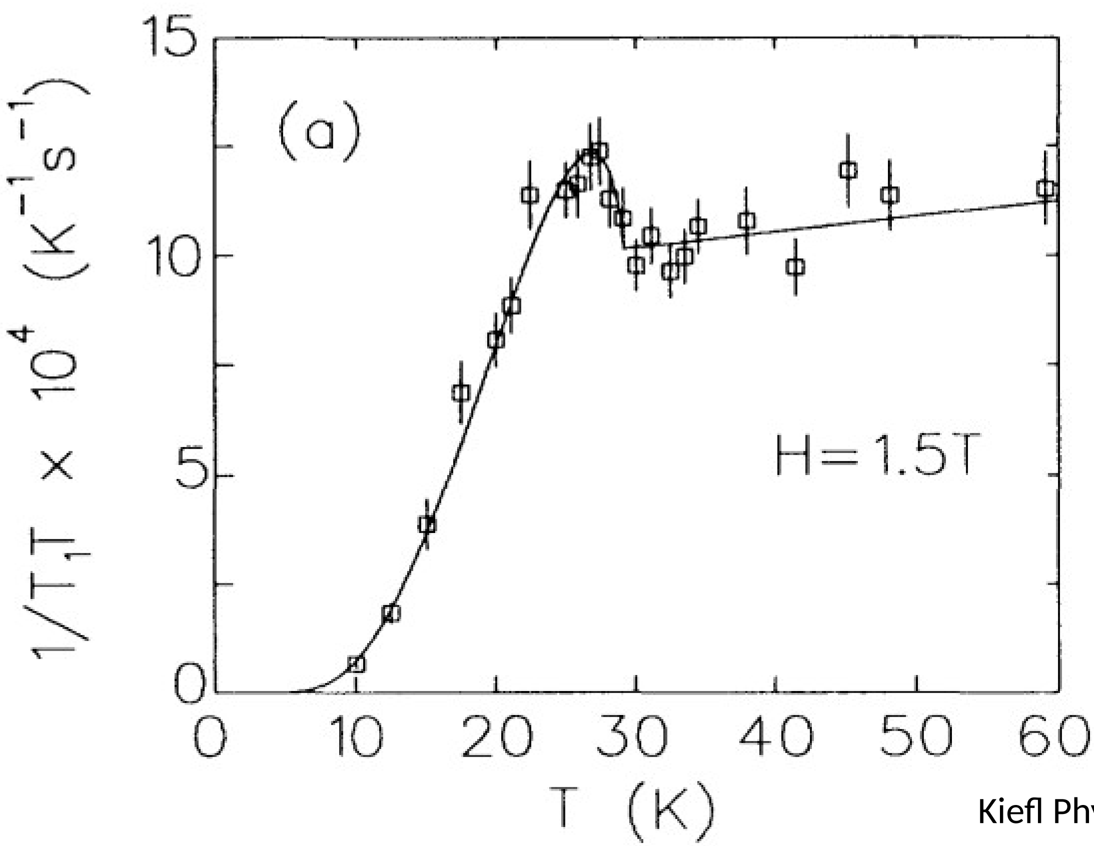
$$C(\epsilon, \Delta(T)) = \frac{1}{2}\left(1 + \frac{\Delta(T)}{E}\right)$$

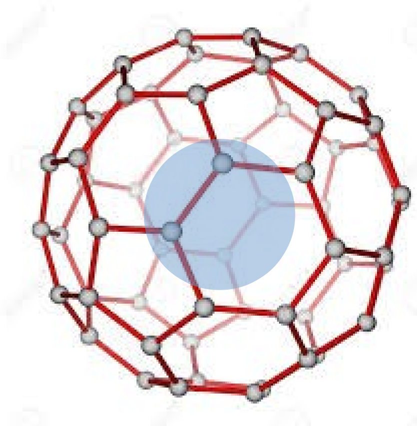
Coherence factor

The coherence peak in Rb_3C_{60} by µSR

$$\frac{1}{T_{1s}} = -\frac{\pi k_B T}{\hbar} A_0^2 \int_{-\epsilon_F}^{\infty} N^2(\epsilon) \frac{\partial f(E)}{\partial \epsilon} C(E,\Delta) d\epsilon$$







$Mu@C_{60}$
T_{1n} very long

Kiefl Phys Rev Lett 70 3987

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A superconductors phase diagram

123 = $YBa_2Cu_3O_{6+x}$
214 = $La_{2-x}Sr_xCuO_4$
2212 = $Bi_2Sr_2CaCu_2O_8$
2223 = $Bi_2Sr_2Ca_2Cu_3O_{10}$

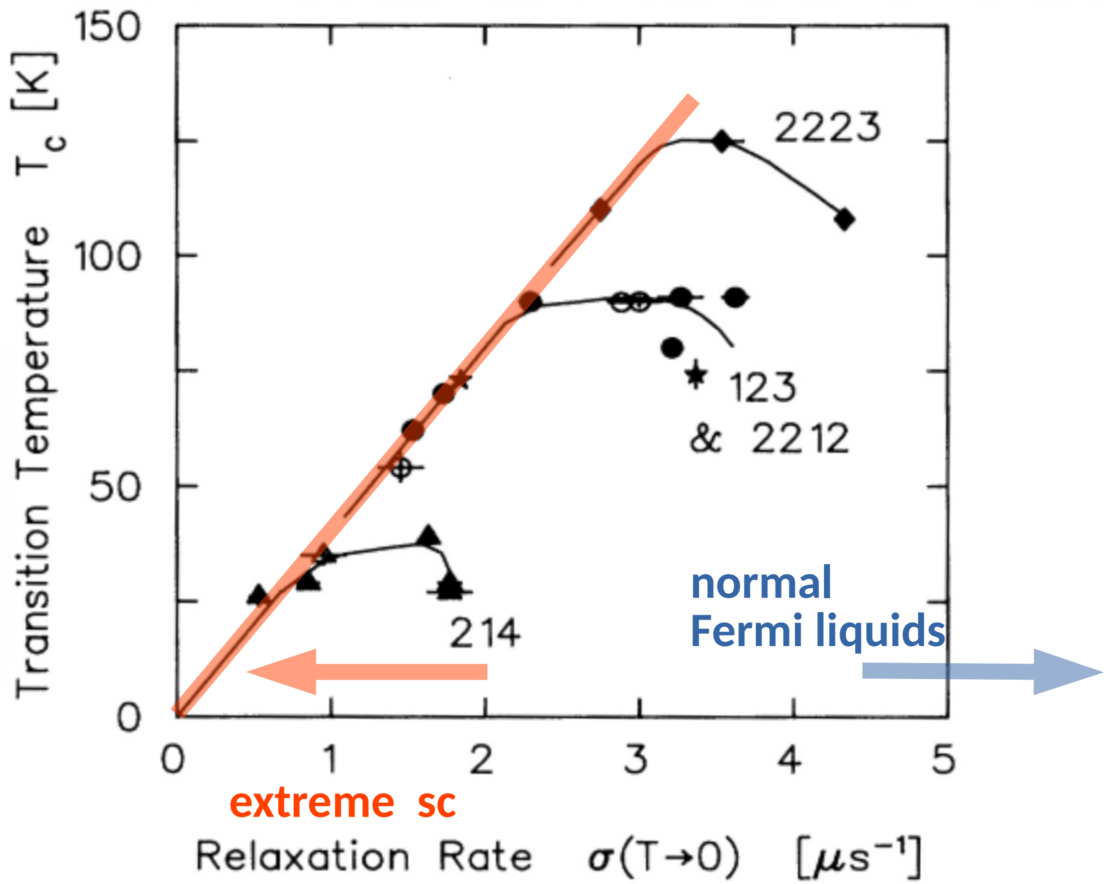
BCS weak coupling

$$T_c \propto \hbar\omega_b e^{-\frac{1}{N(0)\overline{V}}}$$

highly unconventional

$$T_c \propto \frac{n_s}{m}$$

The ur-Uemura plot

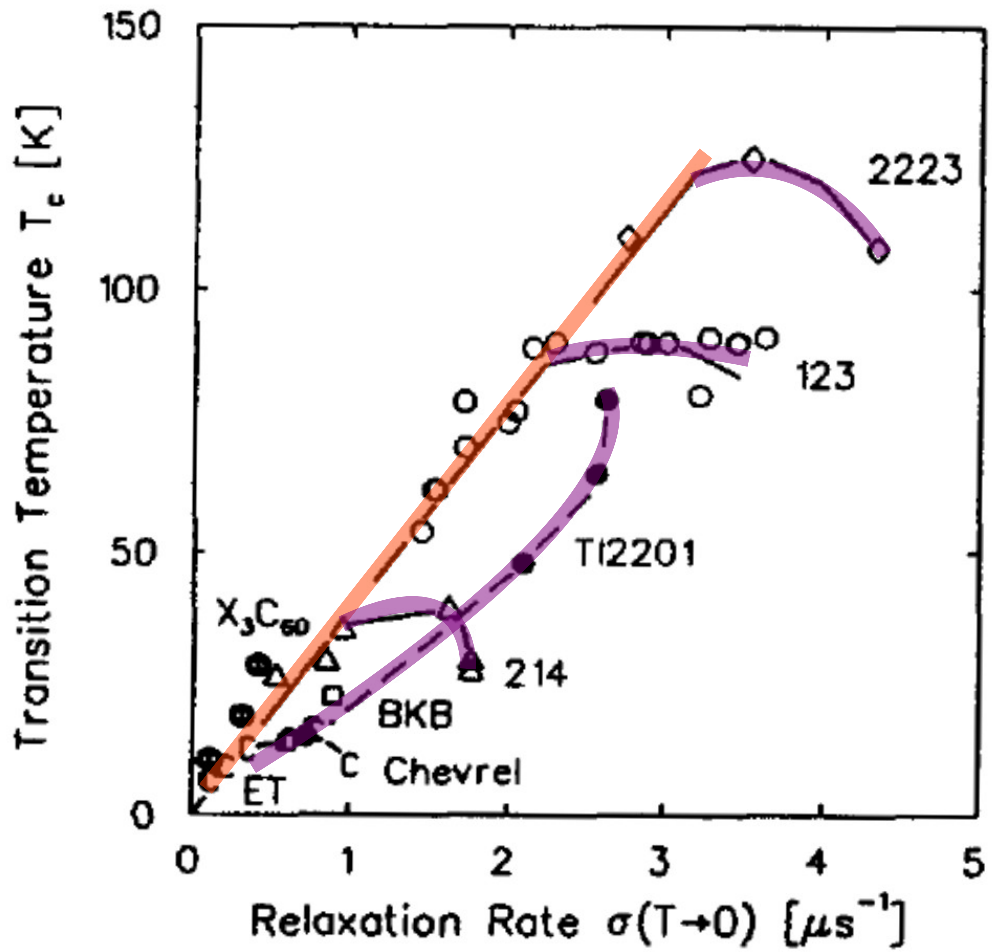


$$\sigma(0) \propto \frac{n_s}{m}$$

Uemura Phys Rev Lett 62 2317 1300 citations (Scholar)

A richer Uemura plot

Uemura Phys Rev Lett 66 2667   700 citations (Scholar)



underdoped cuprates

overdoped

* all extreme type II, no correction for mean free path

highly unconventional

$$T_c \propto \frac{n_s}{m}$$

seems to be a boundary

For 2d $$T_F = \frac{\pi\hbar^2}{2k_B}\frac{n}{m}$$ Does this mean that the boundary is $\propto T_F$?

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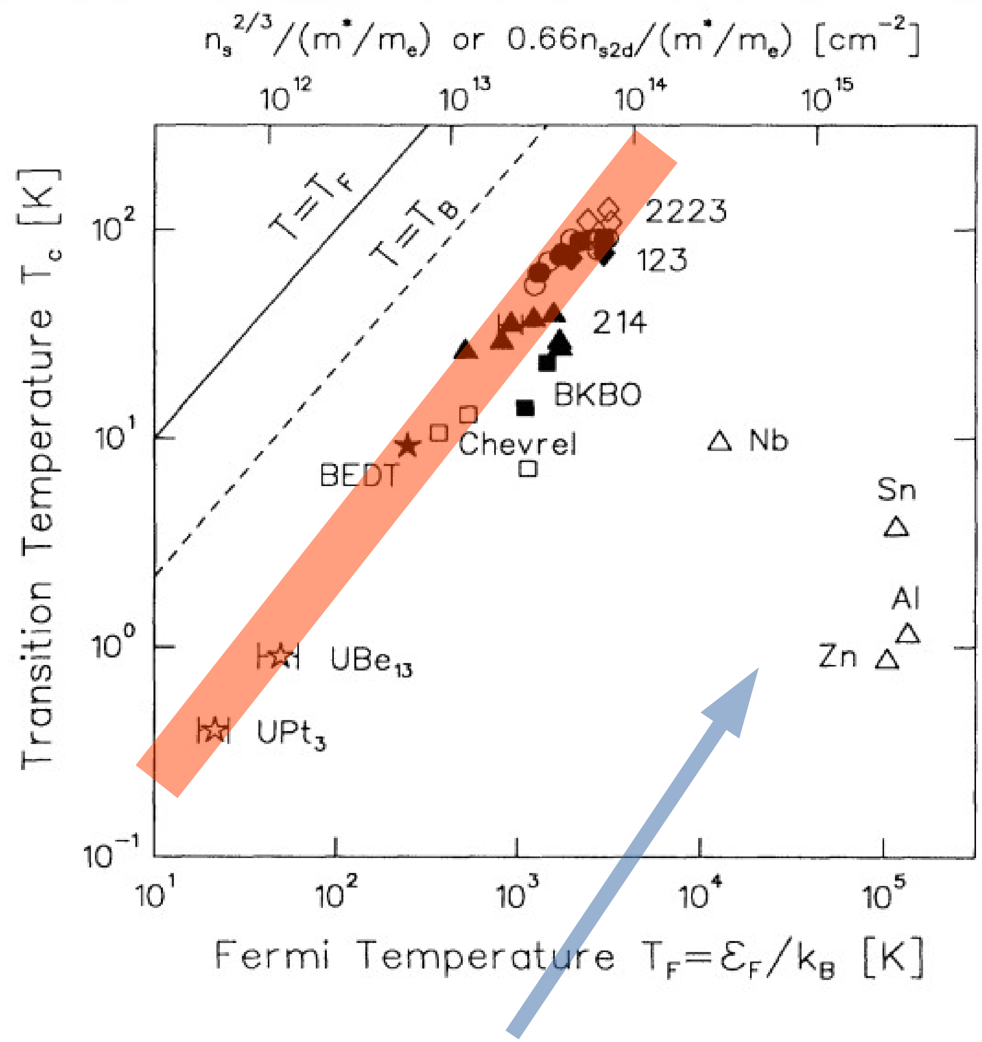
Uemura plot: a possible rationale

If the boundary is T_F we must rescale $\sigma \propto \dfrac{n}{m}$ of 3D materials to get them $\propto T_F$

Use Sommerfeld (specific heat)

$$\gamma = \left[\frac{\pi}{3}\right]^{\frac{2}{3}} \left[\frac{k_B}{\hbar}\right]^2 m n^{\frac{1}{3}}$$

$$T_F = \frac{\hbar^2}{2k_B}\frac{(3\pi^2 n)^{\frac{2}{3}}}{m} \propto \left[\frac{\sigma^3}{\gamma}\right]^{\frac{1}{4}}$$



The distinction with conventional superconductors for very large densities, is evident

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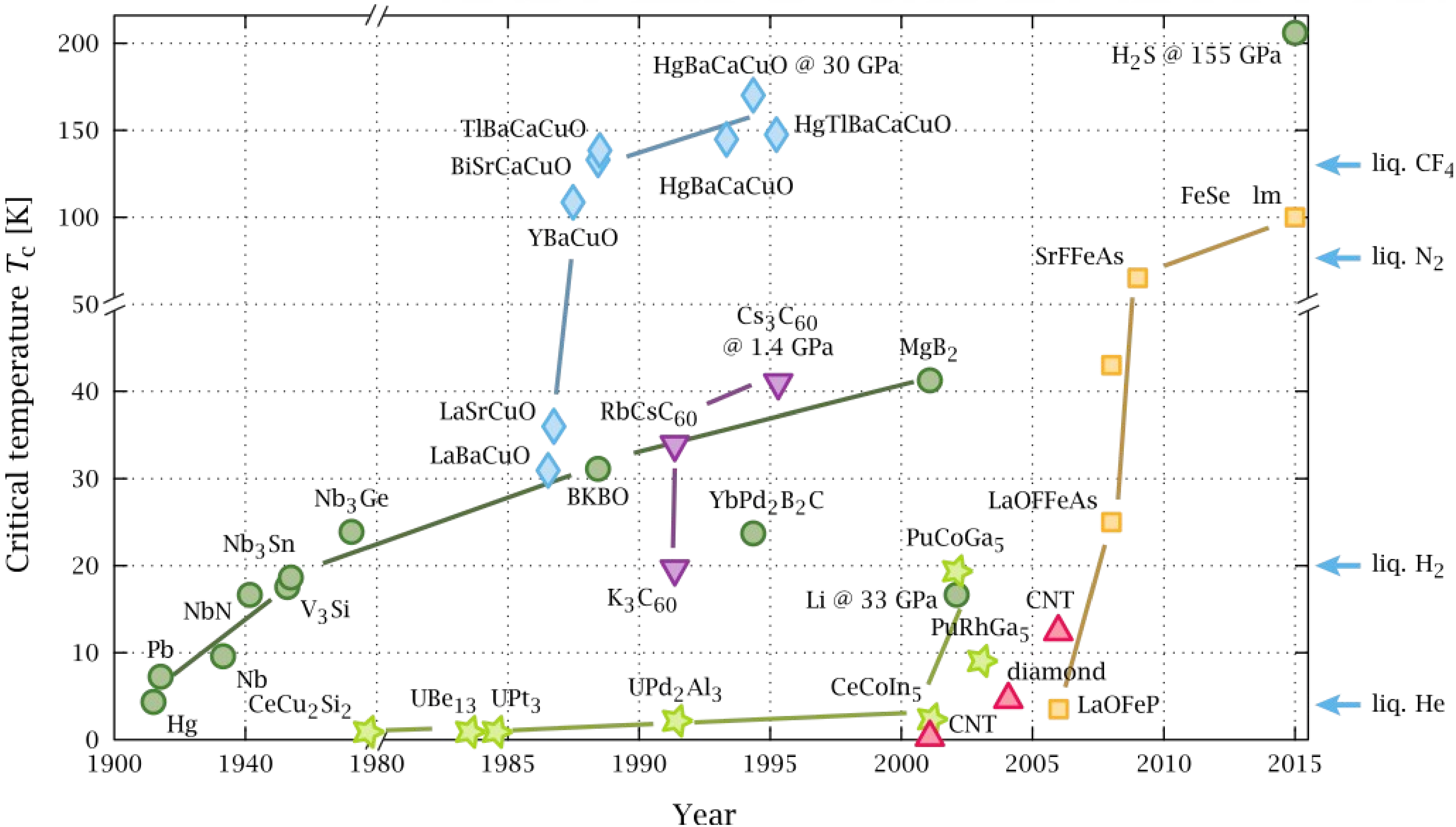
Conventional and unconventional



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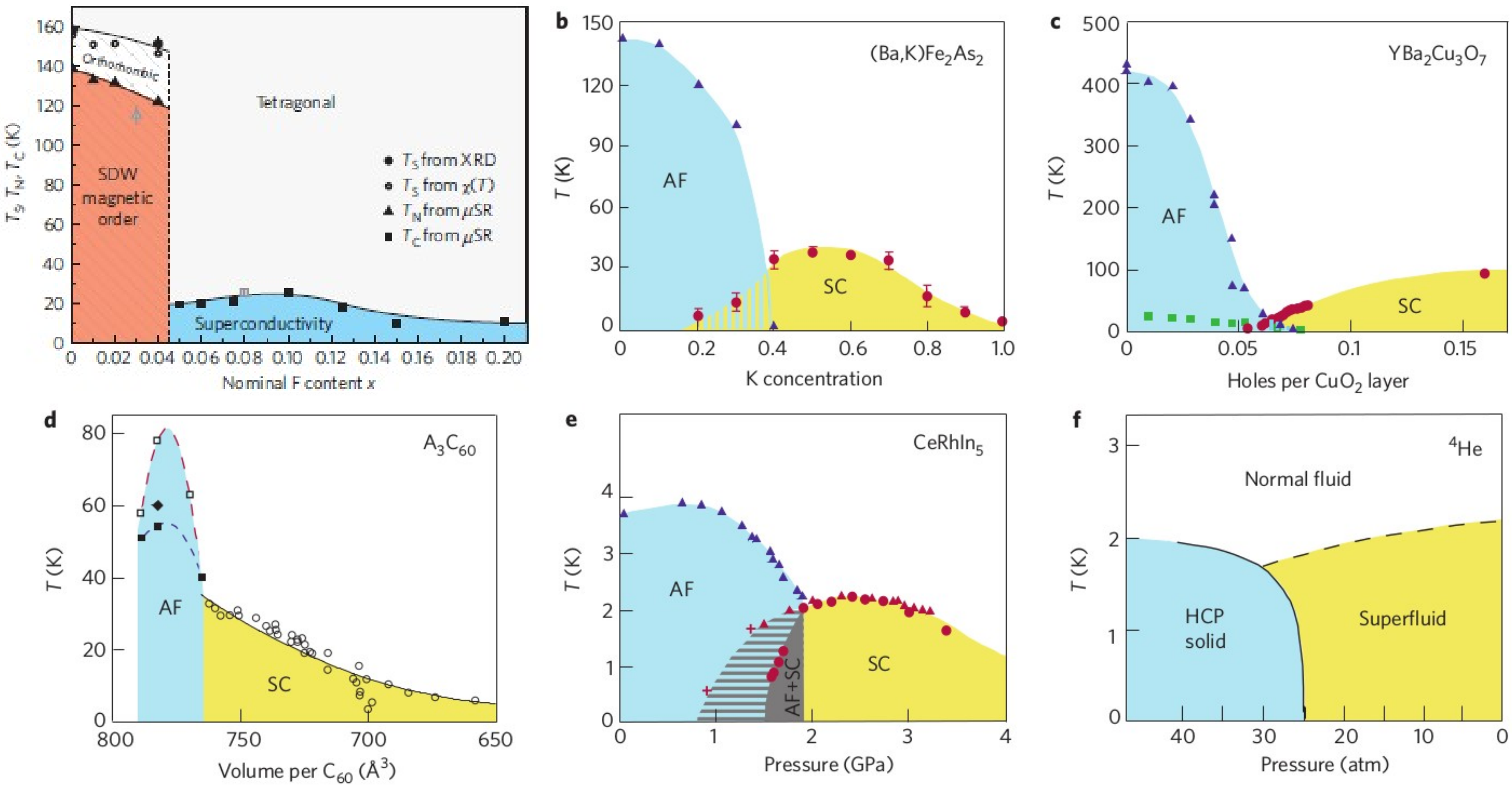
Commonalities among unconventional superconductors

Luetkens Nat Mat 8 305

Sanna Phys Rev Lett 93 207001



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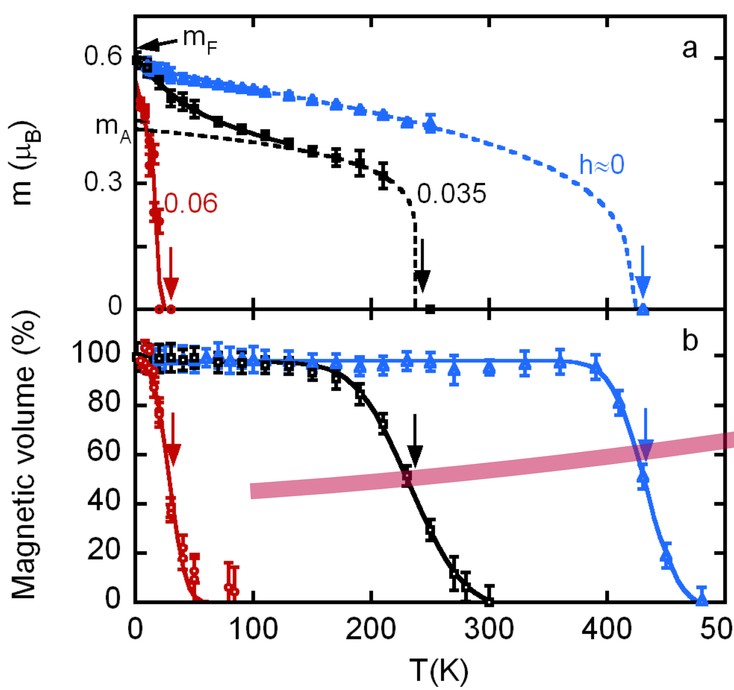
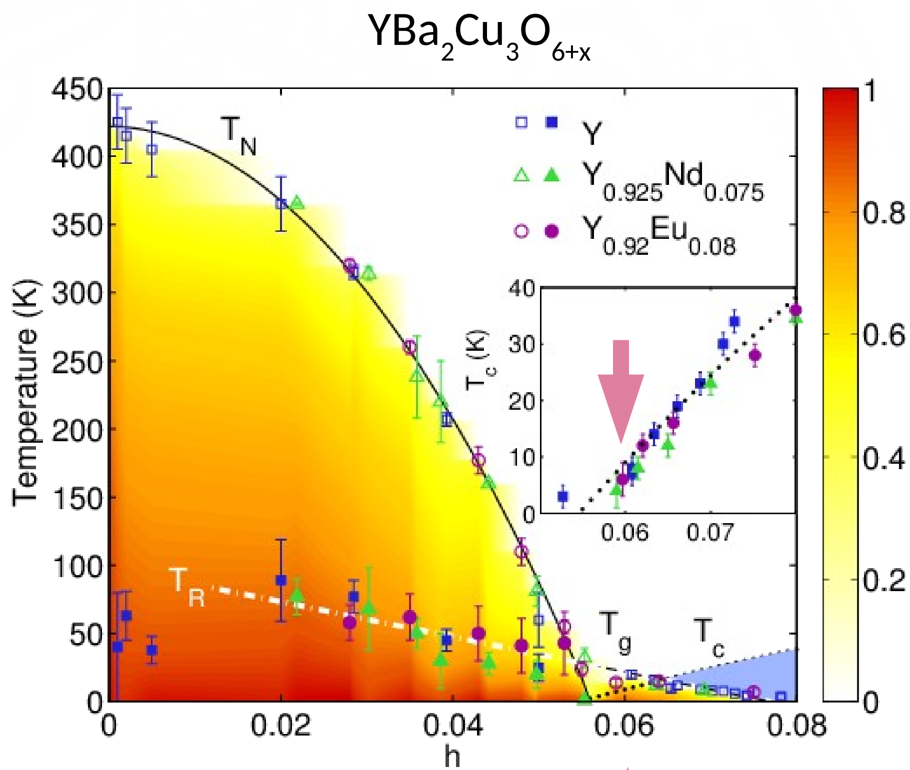
Nanoscopic coexistence

hole content from extensive calibrations
(structure, transport properties)

Samples with a bulk T_c display

- nearly full Cu moment
- full magnetic volume fraction





Coneri Phys Rev B 81 104507

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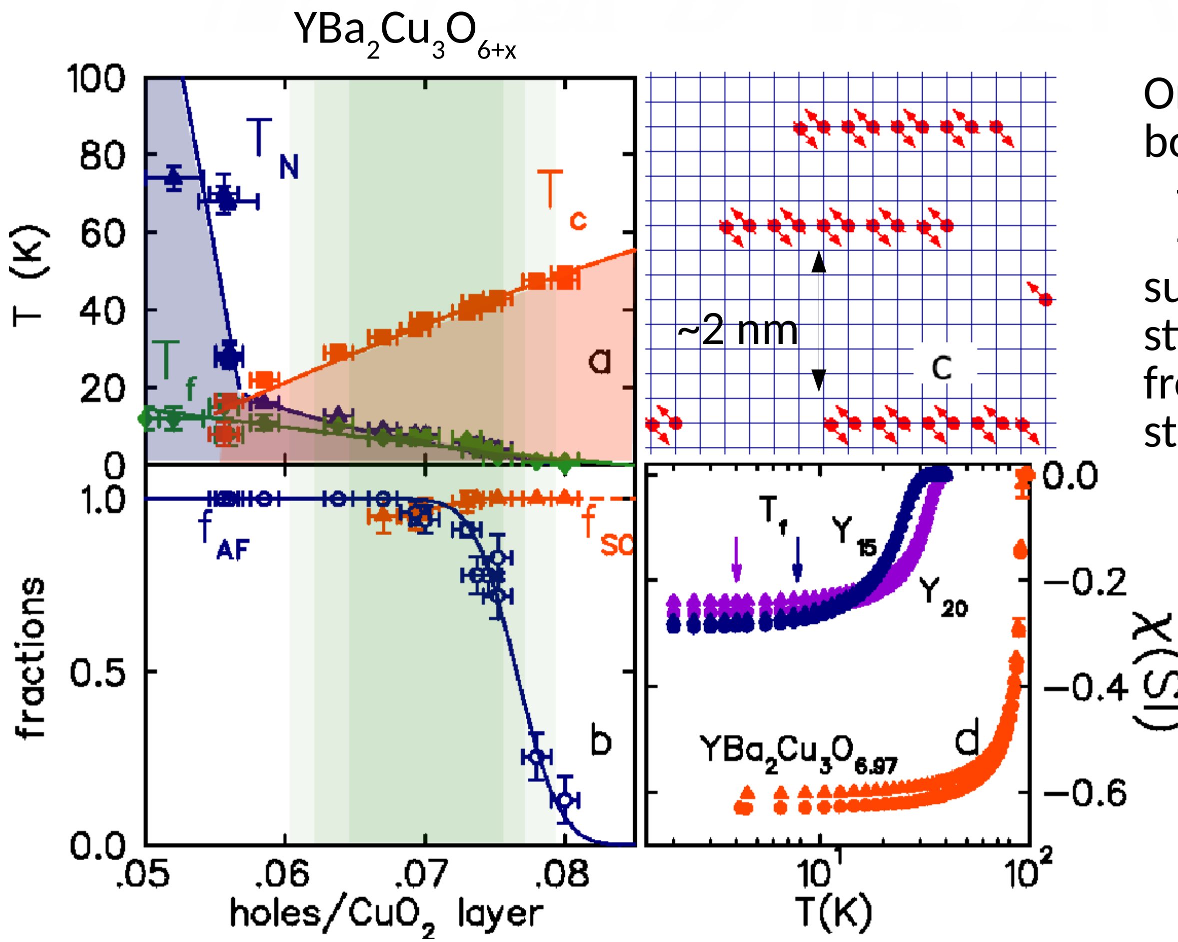
Nanoscopic coexistence



Only possible if the both fractions are
- close to 50%
- few nm across

such that μ in sc volumes still detect fields from nearby magnetic stripes

Here the sc volume fraction and the volume fraction where μ detect local fields are both close to 100%

Sanna Phys Rev Lett 93 207001

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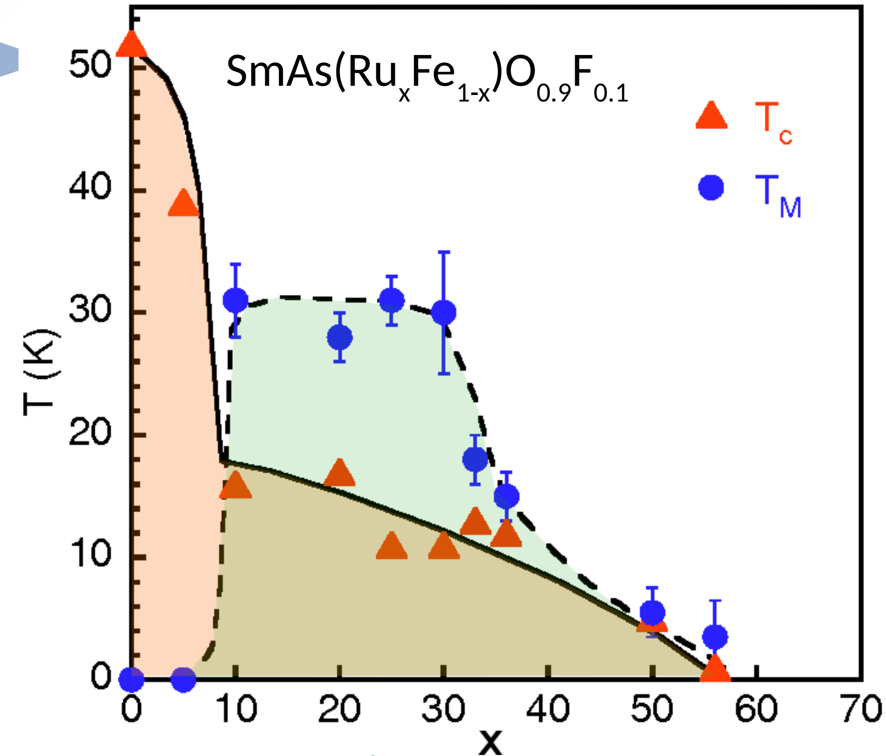
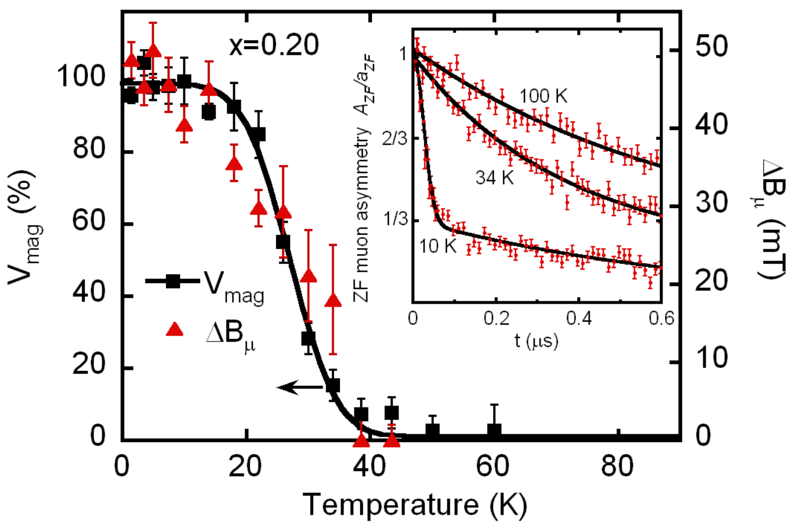
Nanoscopic coexistence



Optimally doped superconductor, T_c = 55 K

Substituting Ru for Fe suppresses superconductivity with a reentrant magnetic phase

Sanna Phys Rev Lett 107 227003



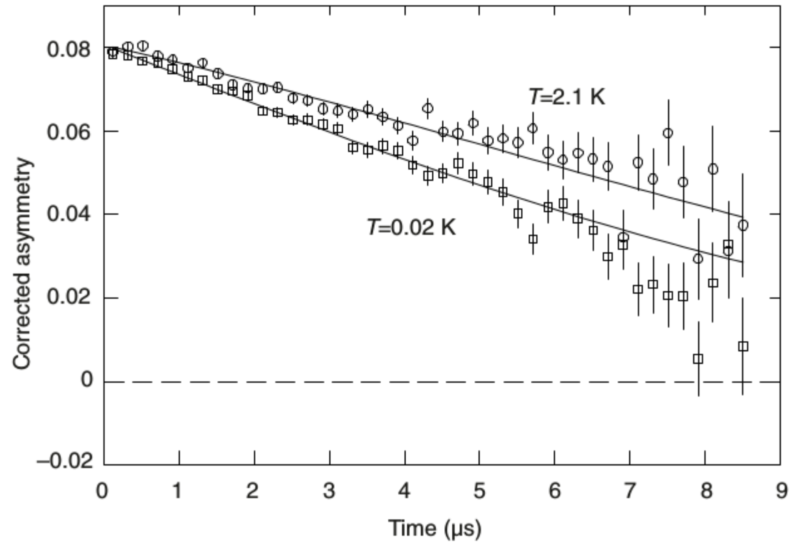
Here the sc volume fraction and the volume fraction where μ detect local fields are both larger than 50%

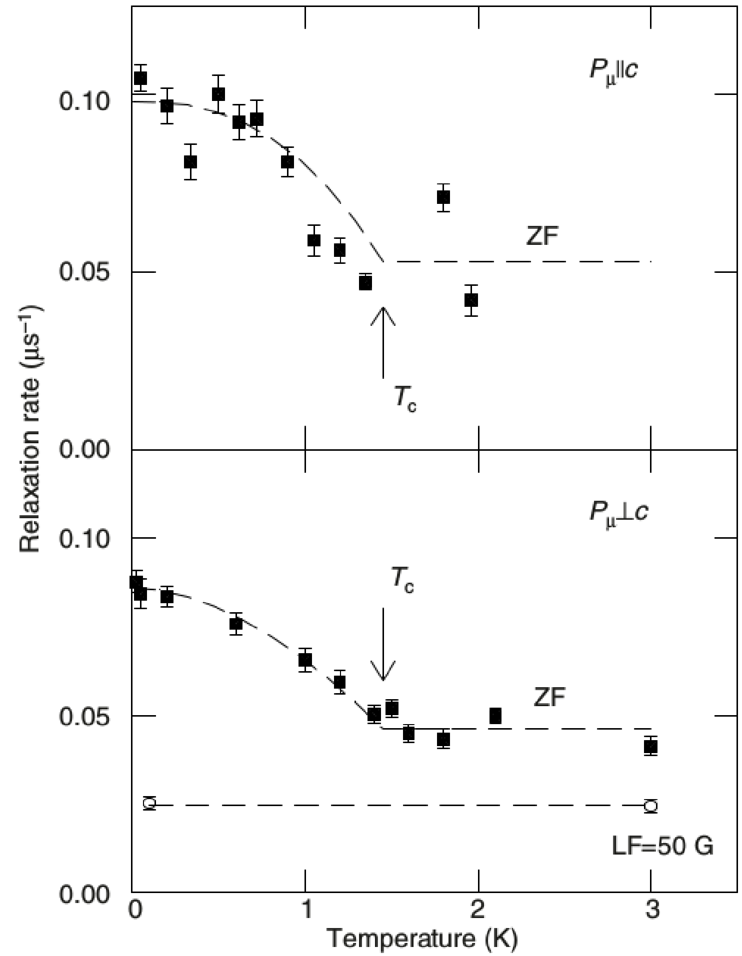
Time Reversal Symmetry Breaking

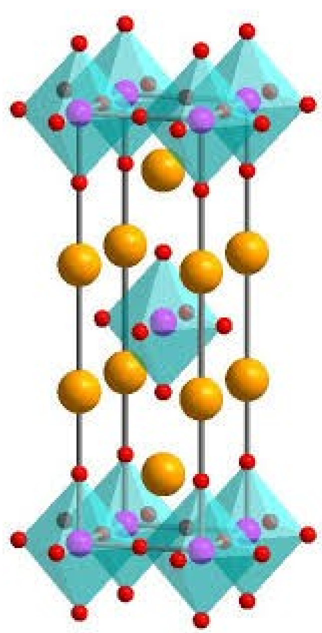
Sr_2RuO_4 p-wave superconductor

Luke Nature 394, 558

Zero field μSR





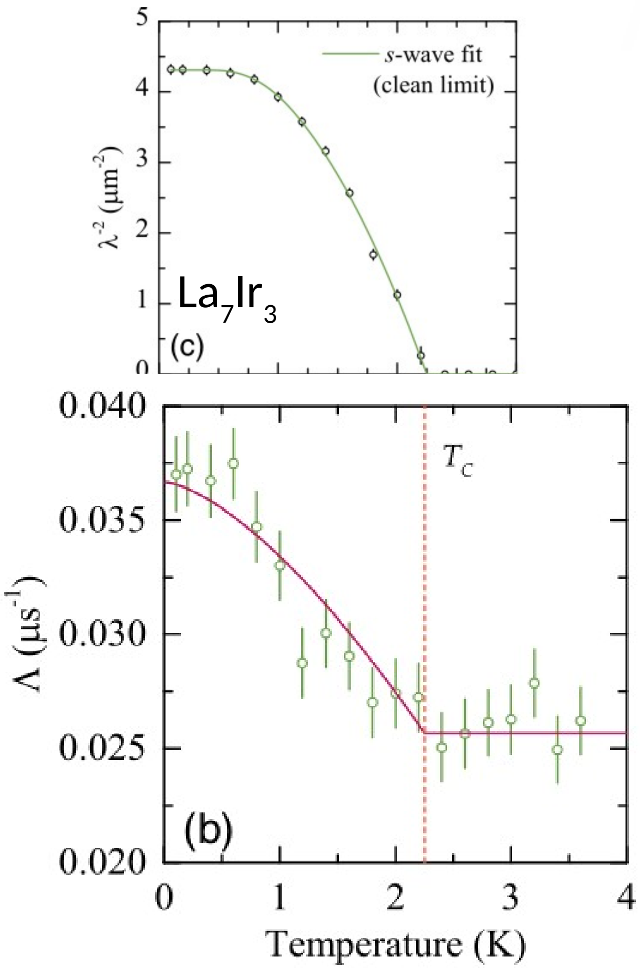


Triplet pairing is compatible with orbital currents

An additional internal field is detected by muons below T_c

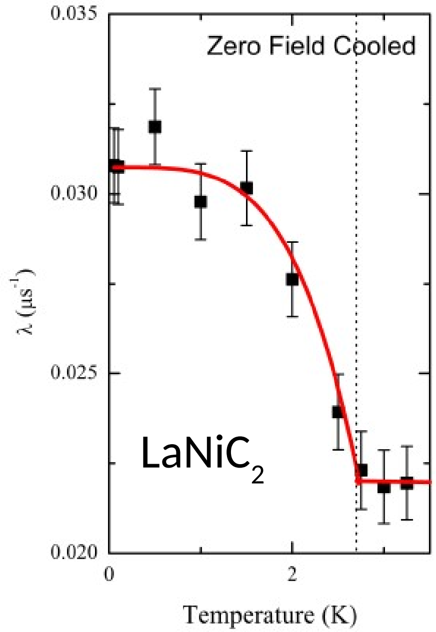
Other techniques: Josephson interferometry, Kerr effect, scanning SQUID microscopy

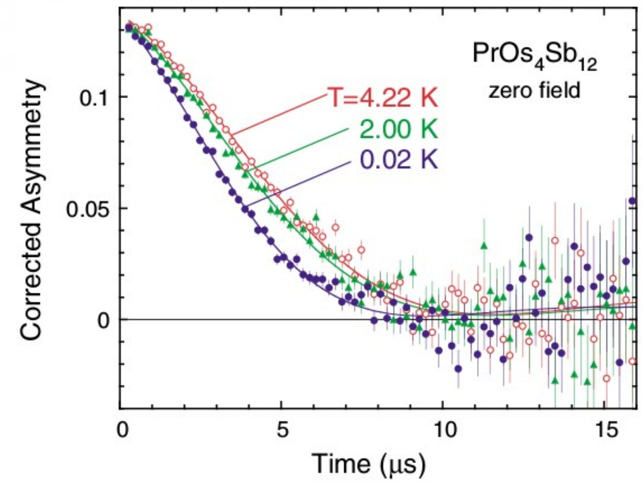
More TRSB

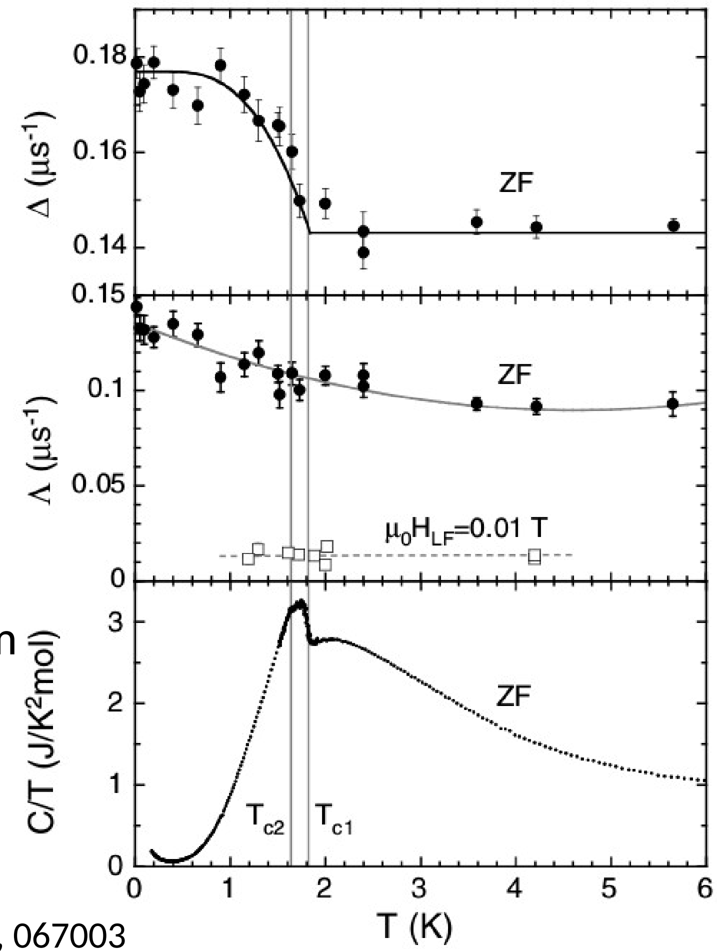


Barker Phys Rev Lett 115 267001

Hillier, Phys. Rev. Lett. 102, 117007







weak additional magnetism is observed also in some non centrosymmetric superconductors

Aoki, Phys. Rev. Lett. 91, 067003

That’s all, thank you

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